Similarity Search

Trees and Relational Databases

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Outline

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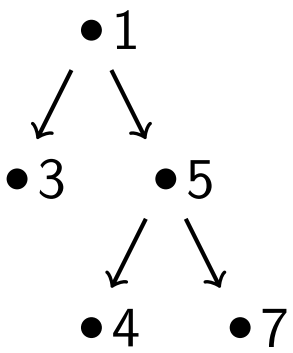
What is a Tree?

- Graph: a pair (N, E) of nodes N and edges E between nodes of N
- Tree: a directed, acyclic graph T
  - that is connected and
  - no node has more than one incoming edge
- Edges: $E(\mathrm{T})$ are the edges of T
  - an edge $(\mathrm{p}, \mathrm{c}) \in E(\mathrm{T})$ is an ordered pair
  - with $\mathrm{p}, \mathrm{c} \in N(\mathrm{T})$
- "Special" Nodes: $N(\mathrm{T})$ are the nodes of T
  - parent/child: $(\mathrm{p}, \mathrm{c}) \in E(T) \Leftrightarrow$ p is the parent of c, c is the child of p
  - siblings: c_1 and c_2 are siblings if they have the same parent node
  - root node: node without parent (no incoming edge)
  - leaf node: node without children (no outgoing edge)
  - fanout: fanout f_v of node v is the number of children of v

Unlabeled Trees

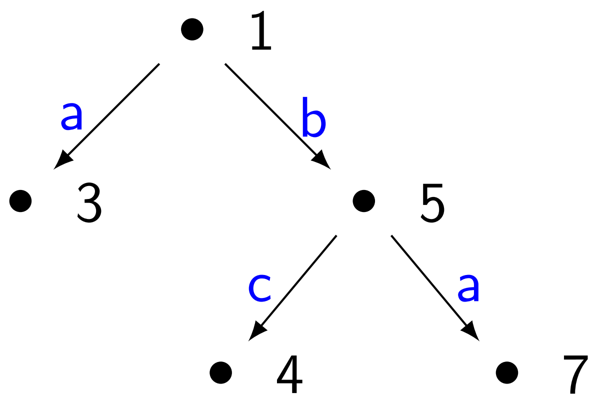
- Unlabeled Tree:
  - the focus is on the structure, not on distinguishing nodes
  - however, we need to distinguish nodes in order to define edges
    $\Rightarrow$ each node v has a unique identifier id(v) within the tree
- Example: $T = (\{1, 3, 5, 4, 7\}, \{(1, 3), (1, 5), (5, 4), (5, 7)\})$



Edge Labeled Trees

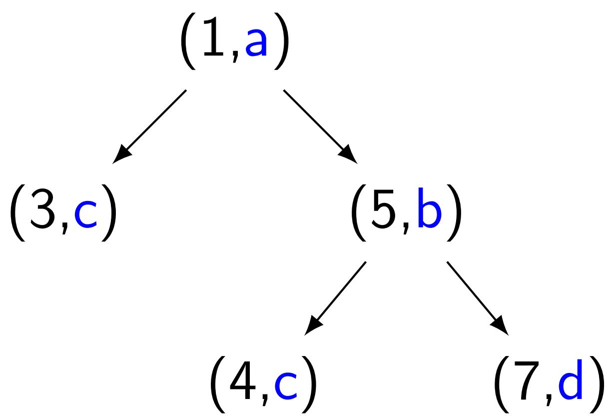
- Edge Labeled Tree:
  - an edge $e \in E(\mathrm{T})$ between nodes a and b is a triple $e = (\mathrm{id}(\mathrm{a}), \mathrm{id}(\mathrm{b}), \lambda(e))$
  - id(a) and id(b) are node IDs
  - $\lambda(e)$ is the edge label (not necessarily unique within the tree)
- Example:

$$\mathrm{T} = (\{1, 3, 5, 4, 7\}, \{(1, 3, \mathtt{a}), (1, 5, \mathtt{b}), (5, 4, \mathtt{c}), (5, 7, \mathtt{a})\})$$



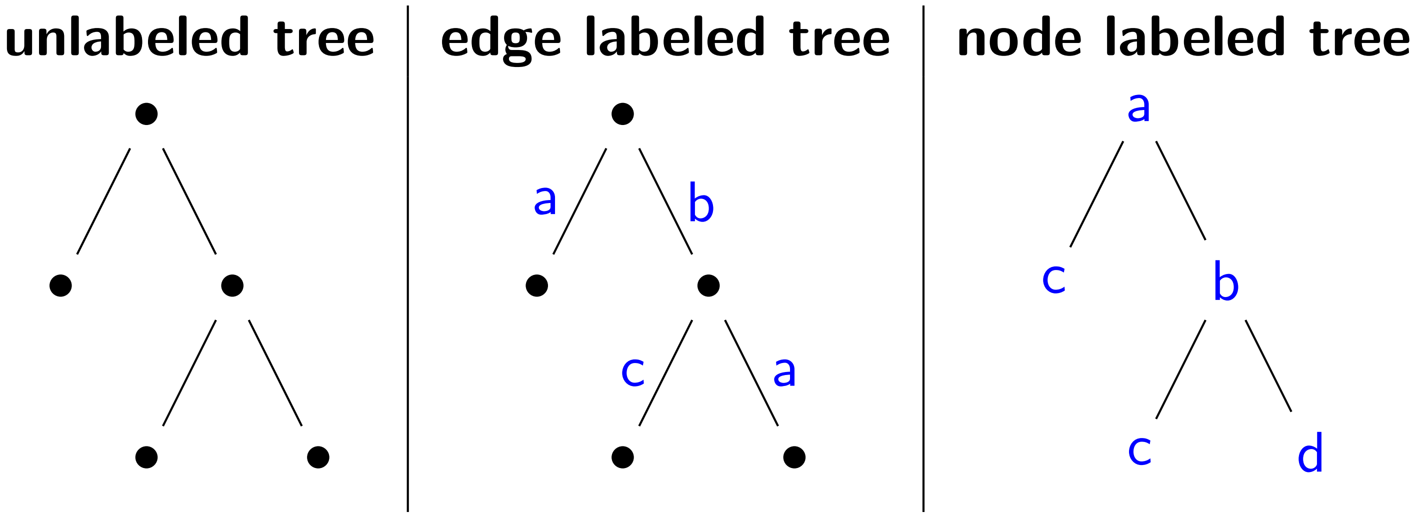
Node Labeled Trees

- Node Labeled Tree:
  - a node $v \in N(T)$ is a pair $(id(v), \lambda(v))$
  - id(v) is unique within the tree
  - label $\lambda(v)$ needs not to be unique
- Intuition:
  - The identifier is the key of the node.
  - The label is the data carried by the node.
- Example: $T = (\{(1, a), (3, c), (5, b), (4, c), (7, d)\},$
  $\{(1, 3), (1, 5), (5, 4), (5, 7)\})$



Notation and Graphical Representation

- Notation:
  - node identifiers: $\mathrm{id}(\mathrm{v}_i) = i$
  - tree identifiers: $\mathrm{T}_1, \mathrm{T}_2, \ldots$
- Graphical representation
  - we omit brackets for (identifier,label)-pairs
  - we (sometimes) omit node identifiers at all
  - we do not show the direction of edges (edges are always directed from root to leave)



Ordered Trees

- Ordered Trees: siblings are ordered
- contiguous siblings $s_1 < s_2$ have no sibling x such that $s_1 < x < s_2$
- c_i is the *i*-th child of p if
  - p is the parent of c_i, and
  - $i = |\{x \in N(T) : (p, x) \in E(T), x \le c_i\}|$
- Example:

Unordered Trees | **Ordered Trees**

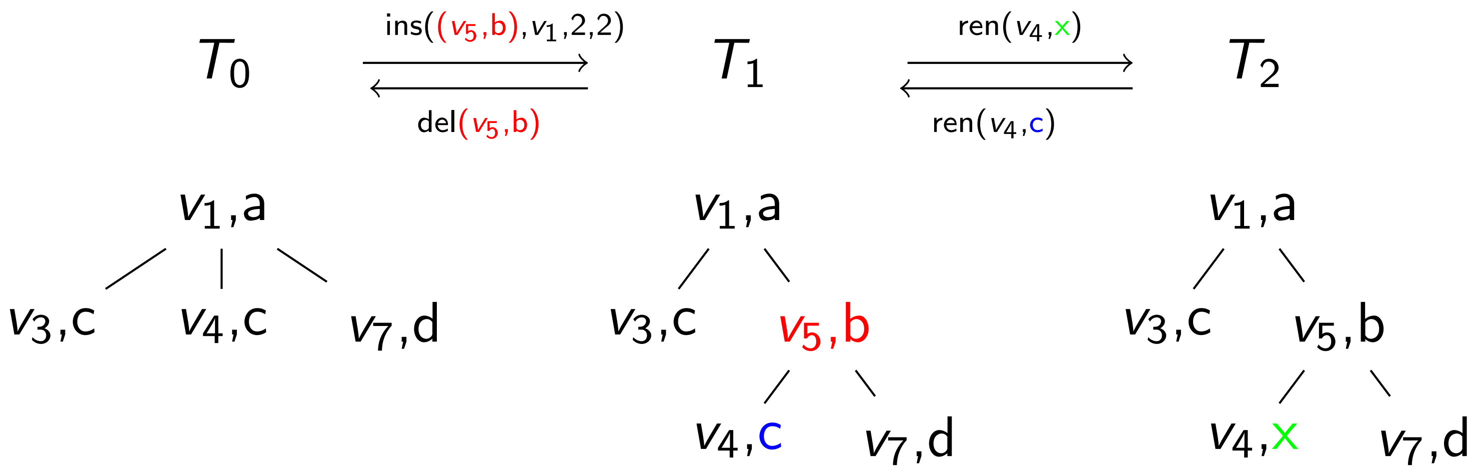
```
     a               a       |      a               a
    /|\             /|\      |     /|\             /|\
   c b d     =     d b c     |    c b d     ≠     d b c
        /\        /\         |         /\        /\
       e  f      f  e        |        e  f      f  e
```

- Note: “ordered” does not necessarily mean “sorted alphabetically”

Edit Operations

- We assume ordered, labeled trees
- Rename node: $ren(\mathrm{v}, l')$
  - change label l of v to $l' \neq l$
- Delete node: $del(\mathrm{v})$ (v is not the root node)
  - remove v
  - connect v's children directly to v's parent node (preserving order)
- Insert node: $ins(\mathrm{v}, \mathrm{p}, k, m)$
  - remove m consecutive children of p, starting with the child at position k, i.e., the children $\mathrm{c}_k, \mathrm{c}_{k+1}, \ldots, \mathrm{c}_{k+m-1}$
  - insert $\mathrm{c}_k, \mathrm{c}_{k+1}, \ldots, \mathrm{c}_{k+m-1}$ as children of the new node v (preserving order)
  - insert new node v as k-th child of p
- Insert and delete are inverse edit operations (i.e., insert undoes delete and vice versa)

Example: Edit Operations



Outline

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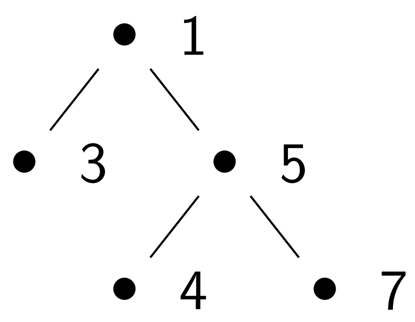
Motivation: Trees and Relational Databases

- Relational Databases:
  - highly developed systems
  - mature storage and querying capabilities
- But: there is a gap between ordered trees and relations
  - relations are sets (no order)
  - relations store tuples (no hierarchy)
- How can we store an (ordered) tree in a relation?

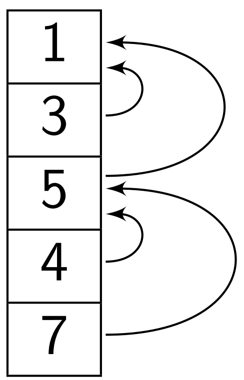
Adjacency List

- Adjacency List:
  - list of nodes
  - each node stores pointer to parent
- Relational Implementation:
  - node is tuple (nid, pid)
  - *nid* the node ID
  - *pid* the node ID of the parent node
- Example:

tree



adjacency list



relational implementatic

nid	pid
1	@
3	1
5	1
4	5
7	5

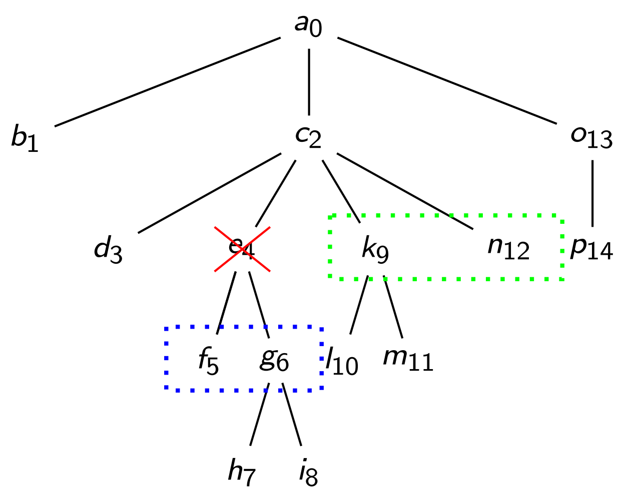
Extending the Adjacency List Model

- Node labeled trees: $(\mathrm{v}, \mathrm{p}, \lambda(\mathrm{v}))$
  - $\mathrm{v}, \mathrm{p} \in N(\mathrm{T})$ are nodes
  - v is a child of p
  - $\lambda(\mathrm{v})$ is the label of v
- Edge labeled trees: $(\mathrm{v}, \mathrm{p}, \lambda((\mathrm{p}, \mathrm{v})))$
  - $\mathrm{v}, \mathrm{p} \in N(\mathrm{T})$ are nodes
  - $(\mathrm{p}, \mathrm{v}) \in E(\mathrm{T})$ is an edge
  - $\lambda((\mathrm{p}, \mathrm{v}))$ is the label of the edge (p, v)
- Ordered trees: $(\mathrm{v}, \mathrm{p}, i)$
  - $\mathrm{v}, \mathrm{p} \in N(\mathrm{T})$ are nodes
  - v is the i-th child of p
- All combinations possible...

Edit Operations with the Adjacency List Encoding

- Tree relation $T(nid, pid, lbl, pos)$
- Rename: $ren(\text{v}, l')$
  - update single tuple $(\text{v}, \text{p}, l, i) \rightarrow (\text{v}, \text{p}, l', i)$
- Delete node: $del(\text{v})$
  - delete single tuple
  - update right siblings and all children of v
- Insert node: $ins(\text{v}, \text{p}, k, m)$
  - insert single tuple
  - update right siblings ($pos \geq k$) and all children of new node v

Example: Delete Node in Adjacency Encoding



nid	pid	pos	lbl
0	-	-	a
1	0	1	b
2	0	2	c
3	2	1	d
~~4~~	~~2~~	~~2~~	~~e~~
5	~~4~~ 2	~~1~~ 2	f
6	~~4~~ 2	~~2~~ 3	g
7	6	1	h
8	6	2	i
9	2	~~3~~ 4	k
10	9	1	l
11	9	2	m
12	2	~~4~~ 5	n
13	0	3	o
14	13	1	p

Update Efficiency

- Worst case: all children of v and of p must be updated
  - $O(f_{max})$ node updates, where f_{max} is the maximum fanout in the tree
  - f_{max} typically small compared to tree size
  - update very efficient
- Implementation hints:
  - unique index on *nid* and on (pid, pos) will speed up queries
  - use `...ORDER BY pos ASC/DESC` in update statement to avoid duplicates

Preorder Traversal

- Preorder: in XML also "document order"
  - visit root
  - traverse subtrees rooted in children (from left to right) in preorder
- Example: preorder $= (\texttt{a}, \texttt{d}, \texttt{f}, \texttt{e}, \texttt{c}, \texttt{b})$

```
    a
   /|\
  d b c
 / \
f   e
```

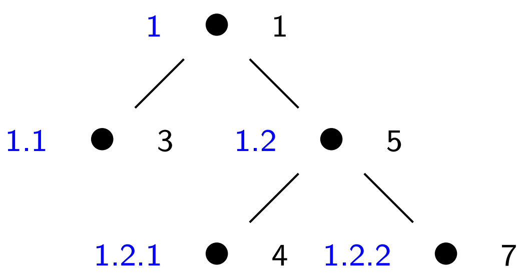
- Implementation:
  - start with root
  - recursively select children of root
- Efficiency:
  - children of all ancestors on recursion-stack
  - $O(n)$ queries for children — very inefficient

Outline

Dewey Encoding

- Dewey Decimal Classification:
  - used in libraries to classify books by topics
  - developed by Melvil Dewey in 1876
- Dewey Encoding[1] [TVB+02]:
  - list of nodes
  - each node stores path from the root
- Example:

tree

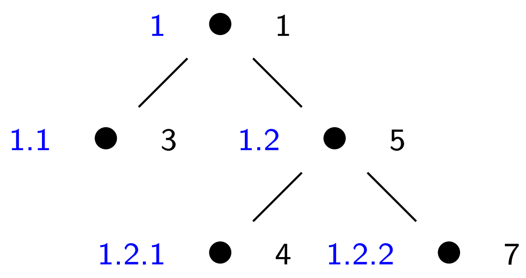


relational implementation

nid	pid
1	1
3	1.1
5	1.2
4	1.2.1
7	1.2.2

[1] also "Edge Enumeration" [Cel04]

About the Dewey Paths



- "$\circ$" concatenates a Dewey path *dp* with an integer *i* (sibling position) e.g., $1.2 \circ 2 = 1.2.2$
- Sort order: $1.2 < 1.3$, $1.1 < 1.1.2$, $1.9 < 1.10$

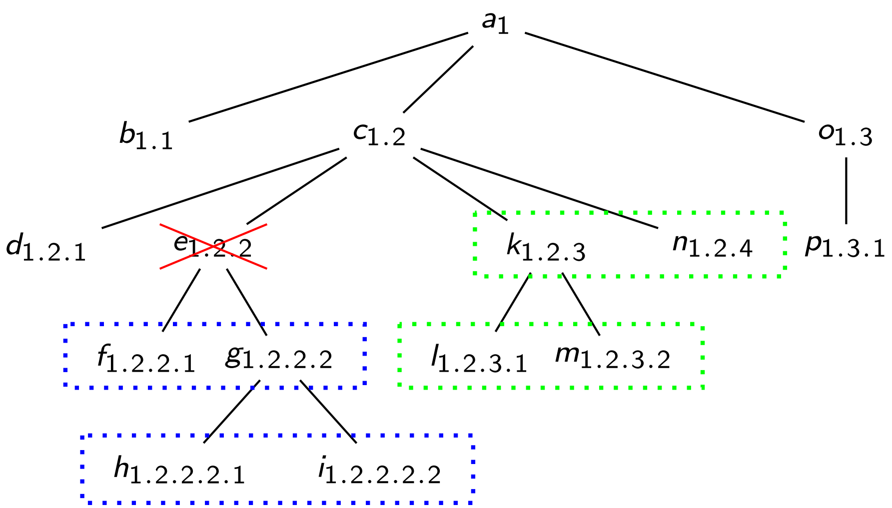
Extending the Dewey Encoding

- Dewey encoding implicitly orders trees!
- Node labeled trees: $(\mathrm{v}, dp, \lambda(\mathrm{v}))$
  - $\mathrm{v} \in N(\mathrm{T})$ is a node ID
  - dp is the Dewey path to v
  - $\lambda(\mathrm{v})$ is the label of v
- Edge labeled trees: $(\mathrm{v}, dp, \lambda)$
  - $\mathrm{v} \in N(\mathrm{T})$ is a node ID
  - dp is the Dewey path to v
  - λ is the label of the edge from the parent of v to v

Edit Operations with the Dewey Encoding

- Tree relation $T(nid, dp, lbl)$
- Rename node: $ren(\text{v}, l')$
  - update single tuple $(\text{v}, dp, l) \rightarrow (\text{v}, dp, l')$
  - no structure updates
- Delete node: $del(\text{v})$
  - remove single tuple $(\text{v}, dp_{\text{v}}, l)$
  - update nodes with $dp > dp_{\text{v}}$ (descendants of v and descendants v's right-hand siblings)
- Insert node: $ins(\text{v}, \text{p}, k, m)$
  - update nodes with $dp \geq dp(\text{p}) \circ k$ (children of p at position k or larger, and all their descendants)
  - insert single tuple $(\text{v}, dp(\text{p}) \circ k, \lambda(\text{v}))$
- Efficiency:
  - $O(n)$ in the worst case (insert/delete leftmost child of root node)
  - better for nodes with (i) few descendants and (ii) few right siblings
  - $O(1)$ for lonely leaf child of a node

Example: Delete Node in Dewey Encoding



nid	dp	lbl
0	1	a
1	1.1	b
2	1.2	c
3	1.2.1	d
~~4~~	~~1.2.2~~	~~e~~
5	~~1.2.2.1~~ 1.2.2	f
6	~~1.2.2.2~~ 1.2.3	g
7	~~1.2.2.2.1~~ 1.2.3.1	h
8	~~1.2.2.2.2~~ 1.2.3.2	i
9	~~1.2.3~~ 1.2.4	k
10	~~1.2.3.1~~ 1.2.4.1	l
11	~~1.2.3.2~~ 1.2.4.2	m
12	~~1.2.4~~ 1.2.5	n
13	1.3	o
14	1.3.1	p

Preorder

- Tree relation $T(nid, dp, lbl)$
- Implementation:
  - sort by attribute *dp*
  - result is preorder traversal
- Efficiency:
  - single query with sort on string attribute
  - efficient (especially with index on *dp*)

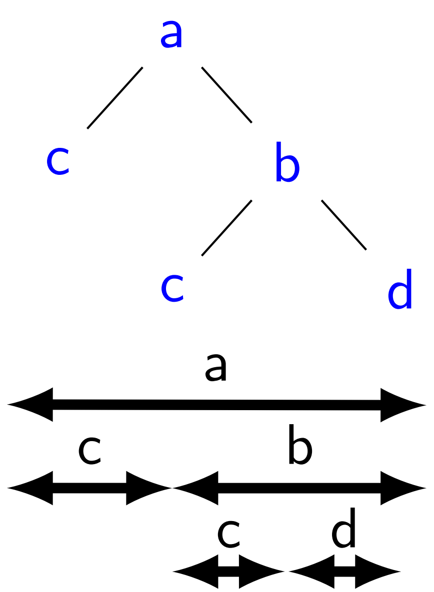
Implementation: Storing the Dewey Path

- Goals:
  - minimize space overhead for Dewey path *dp*
  - sorting Dewey path should result in preorder traversal
- Separator character: e.g., `1.2.5`, `1.17`
  - overhead: small (separator char)
  - sorting: natural sort order not consistent with preorder (`1.2.5` > `1.17`)
- Fixed length: e.g., 0001 0002 0005, 0001 0017
  - overhead: large (small and large numbers require same space)
  - sorting: sort order ok
- Variable length encoding (UTF-8):
  - UTF-8: **1 byte:** $0 \ldots (2^7 - 1)$, **2 bytes:** $2^7 \ldots (2^{11} - 1)$, etc.
  - overhead: small space overhead
  - sorting: sort order ok (supported by many databases, e.g. PostgreSQL)

Outline

Interval Encoding [DTCÖ03, ABG05]

- Idea: Parent “contains” children, like interval contains other intervals
- Example:



- Interval Encoding:
  - assign numbers to interval start and end points
  - store interval start and end point with each node

Interval Encoding

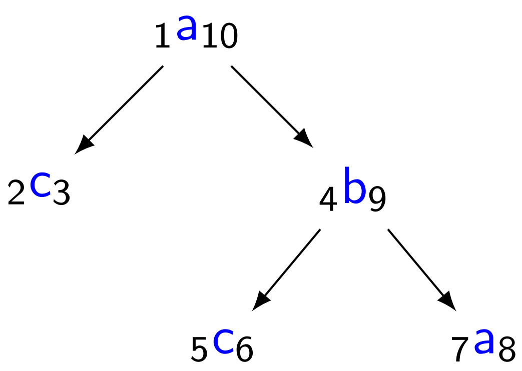
Definition (Interval Encoding)

An *interval encoding* of a tree is a relation T that for each node v of the tree contains a tuple $(\lambda(\mathrm{v}), \mathit{lft}, \mathit{rgt})$; $\lambda(\mathrm{v})$ is the label of v, *lft* and *rgt* are the endpoints of the interval representing the node. *lft* and *rgt* are constrained as follows:

- $\mathit{lft} < \mathit{rgt}$ for all $(\mathit{lbl}, \mathit{lft}, \mathit{rgt}) \in T$,
- $\mathit{lft}_{\mathrm{a}} < \mathit{lft}_{\mathrm{d}}$ and $\mathit{rgt}_{\mathrm{a}} > \mathit{rgt}_{\mathrm{d}}$ if node a is an ancestor of d, and $(\lambda(\mathrm{a}), \mathit{lft}_{\mathrm{a}}, \mathit{rgt}_{\mathrm{a}}) \in T$, and $(\lambda(\mathrm{d}), \mathit{lft}_{\mathrm{d}}, \mathit{rgt}_{\mathrm{d}}) \in T$,
- $\mathit{rgt}_{\mathrm{v}} < \mathit{lft}_{\mathrm{w}}$ if node v is a left sibling of node w, and $(\lambda(\mathrm{v}), \mathit{lft}_{\mathrm{v}}, \mathit{rgt}_{\mathrm{v}}) \in T$, and $(\lambda(\mathrm{w}), \mathit{lft}_{\mathrm{w}}, \mathit{rgt}_{\mathrm{w}}) \in T$,

Example

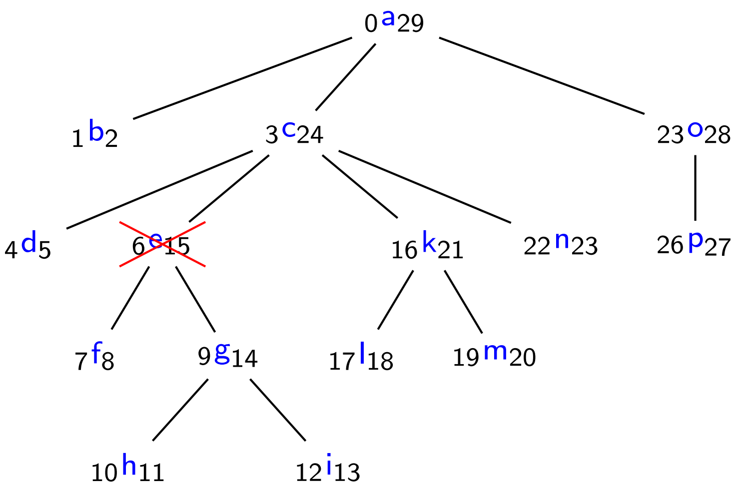
- Example algorithm for a valid interval encoding:
  - traverse tree in preorder
  - use an incremental counter
  - assign left interval value *lft* when node is first visited
  - assign right interval value *rgt* when node is last visited



Edit Operations with the Interval Encoding

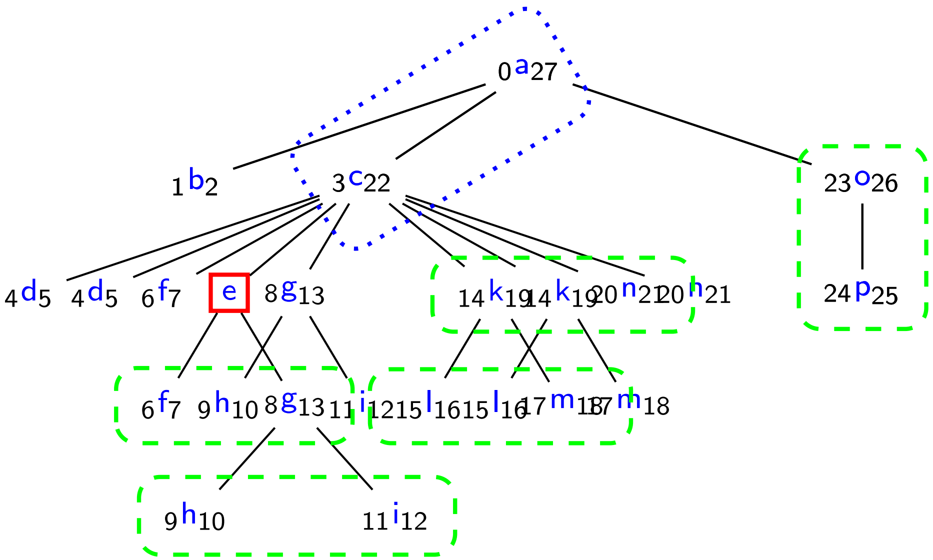
- Tree relation $T(id, lbl, lft, rgt)$
- Rename node: $ren(\text{v}, l')$
  - update single tuple $(\text{id}(\text{v}), l, L, R) \rightarrow (\text{id}(\text{v}), l', L, R)$
  - no structure updates
- Delete node: $del(\text{v})$
  - remove single tuple $(\text{id}(\text{v}), l, L, R)$
  - remaining tree is valid and correct
- Insert node: $ins(\text{v}, \text{p}, k, m)$
  - find left and right interval values L and R
  - if values not free, update ancestors and nodes following in preorder
  - insert single tuple $(\text{id}(\text{v}), \lambda(\text{v}), L, R)$
- Efficiency:
  - rename and delete are very efficient (constant time)!
  - insert may be $O(n)$ in worst case (inefficient)
  - sparse numbering reduces number of updates for insert

Example: Delete Node in Interval Encoding



nid	lbl	lft	rgt
0	a	0	29
1	b	1	2
2	c	3	24
3	d	4	5
~~4~~	~~e~~	~~6~~	~~15~~
5	f	7	8
6	g	9	14
7	h	10	11
8	i	12	13
9	k	16	21
10	l	17	18
11	m	19	20
12	n	22	23
13	o	25	28
14	p	26	27

Example: Insert Node in Interval Encoding



nid	lbl	lft	rgt
0	a	0	~~27~~ 29
1	b	1	2
2	c	3	~~22~~ 24
3	d	4	5
5	f	~~6~~ 7	~~7~~ 8
6	g	~~8~~ 9	~~13~~ 14
7	h	~~9~~ 10	~~10~~ 11
8	i	~~11~~ 12	~~12~~ 13
9	k	~~14~~ 16	~~19~~ 21
10	l	~~15~~ 17	~~16~~ 18
11	m	~~17~~ 19	~~18~~ 20
12	n	~~20~~ 22	~~21~~ 23
13	o	~~23~~ 25	~~26~~ 28
14	p	~~24~~ 26	~~25~~ 27
4	e	6	15

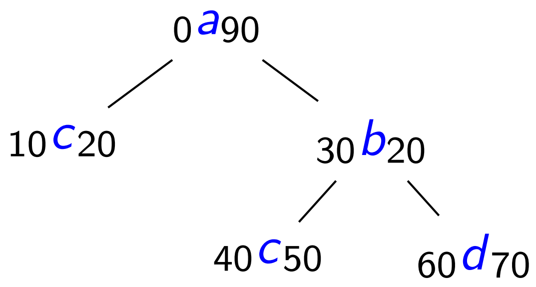
- Insert new node with label e: ins((4, e), 2, 2, 3)
  - update the ancestors of the new node
  - update the nodes following the new node in preorder
  - insert single tuple

Preorder

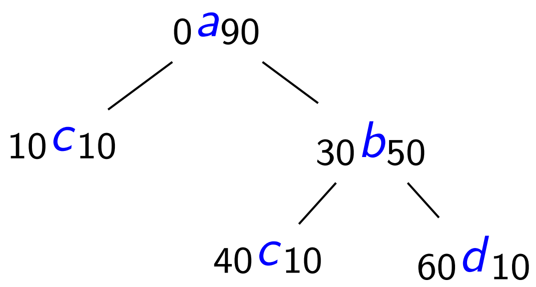
- Tree relation $T(id, lbl, lft, rgt)$
- Implementation:
  - sort by attribute *lft*
  - result is preorder traversal
- Efficiency:
  - single query with sort on integer attribute
  - very efficient (especially with index on *lft*)

Improving Insert/Delete Performance: Sparse Numbering

- Interval Encoding with sparse numbering:
  - leave numbers free for future insert
  - avoids global reordering until gaps are filled
  - node deletions re-open gaps
- Example:



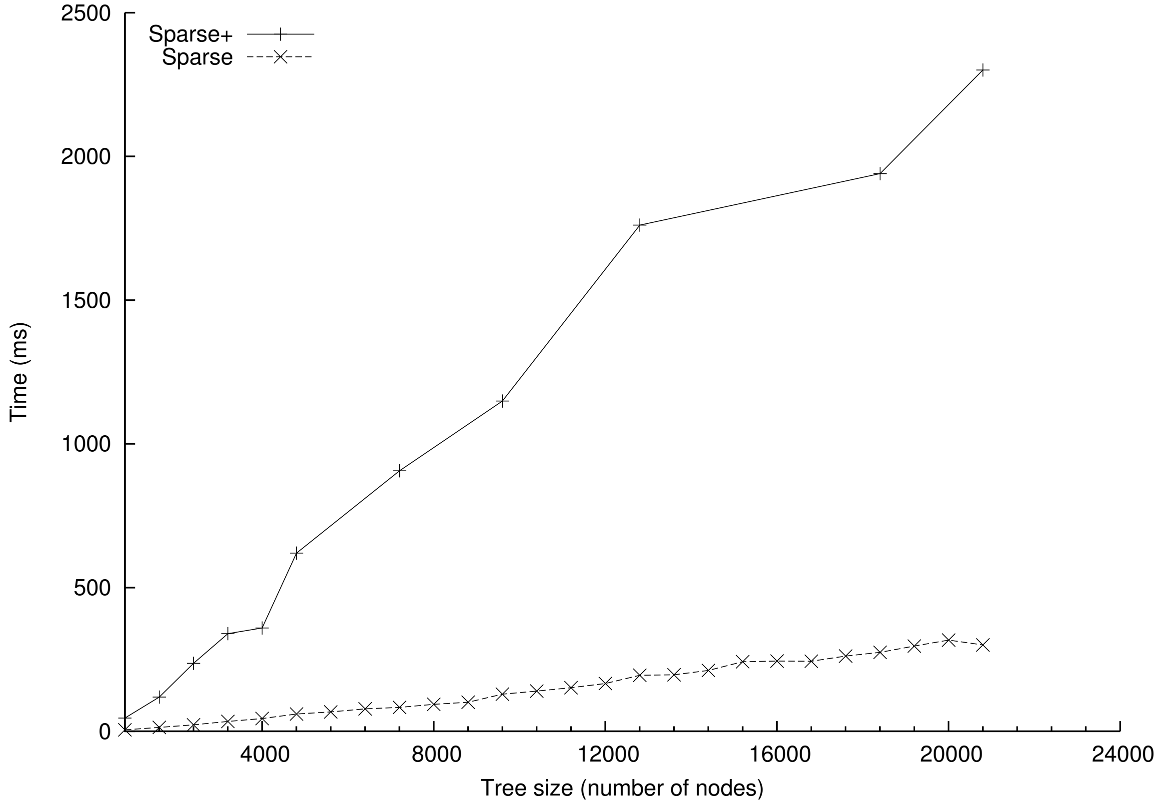
- Note: Floating-point values do not solve the problem!
- Sparse numbering using $(order, size)$-pairs [LM01]:
  - store node position as $(order, size)$-pair
  - $order$ corresponds to left interval value
  - $order + size$ corresponds to right interval value
- Example:



Node Insertion: How To Deal with Full Gaps?

- Inserting a node:
  - a) find the correct gap(s) in the tree
  - b) if the/each gap is large enough: insert new node
  - c) otherwise: ...?
- Solution 1: shift left/right values until new node fits
  - cheapest way for inserting a single node
  - but: only a small number of gaps are opened
- Solution 2: reset all gaps
  - more expensive than shifting
  - but: happens less frequently because all gaps in the tree are opened
- Shifting or resetting gaps are called "hard updates"

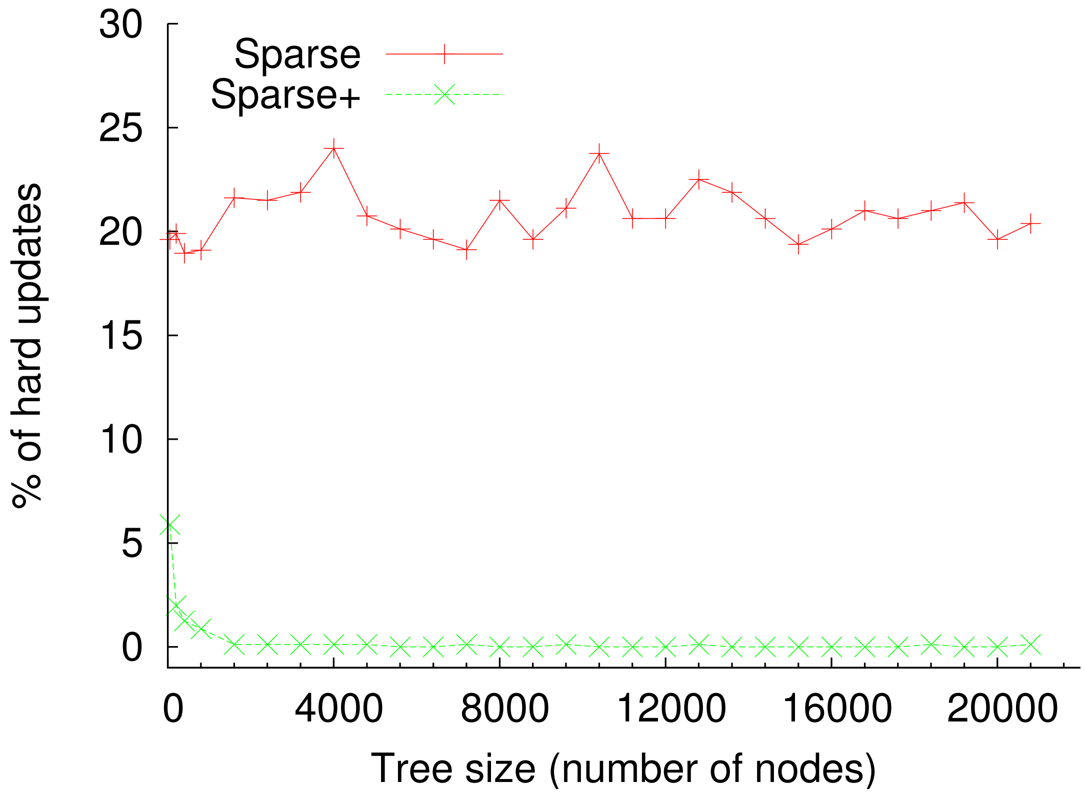
Shifting Gaps is Cheaper than Resetting All Gaps



(Graph from [Dag08])

- Runtime of shifting and resetting:
  - “Sparse+”: resets all gaps
  - “Sparse”: shifts gaps

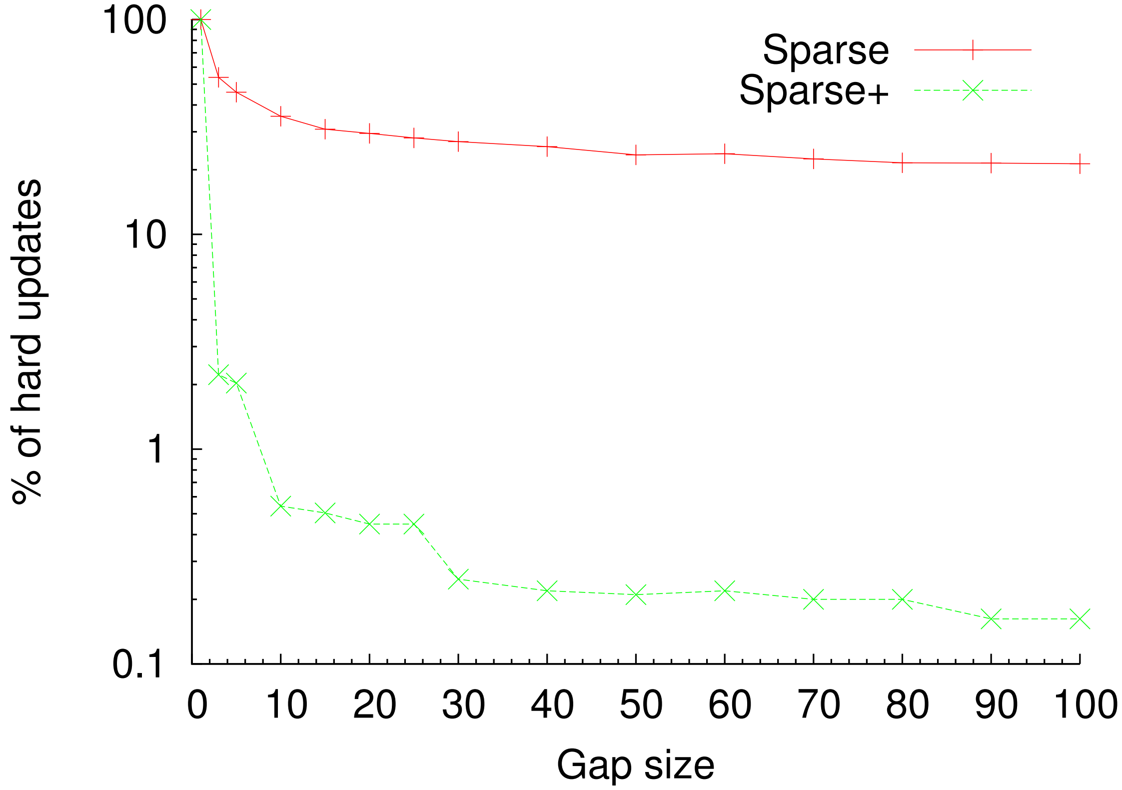
How Often Do We Need a Hard Update



(Graph from [Dag08])

- Average number of hard updates when a new node is inserted (gap size 100):
  - "Sparse+": resets all gaps
  - "Sparse": shifts gaps

Impact of the Gap Size

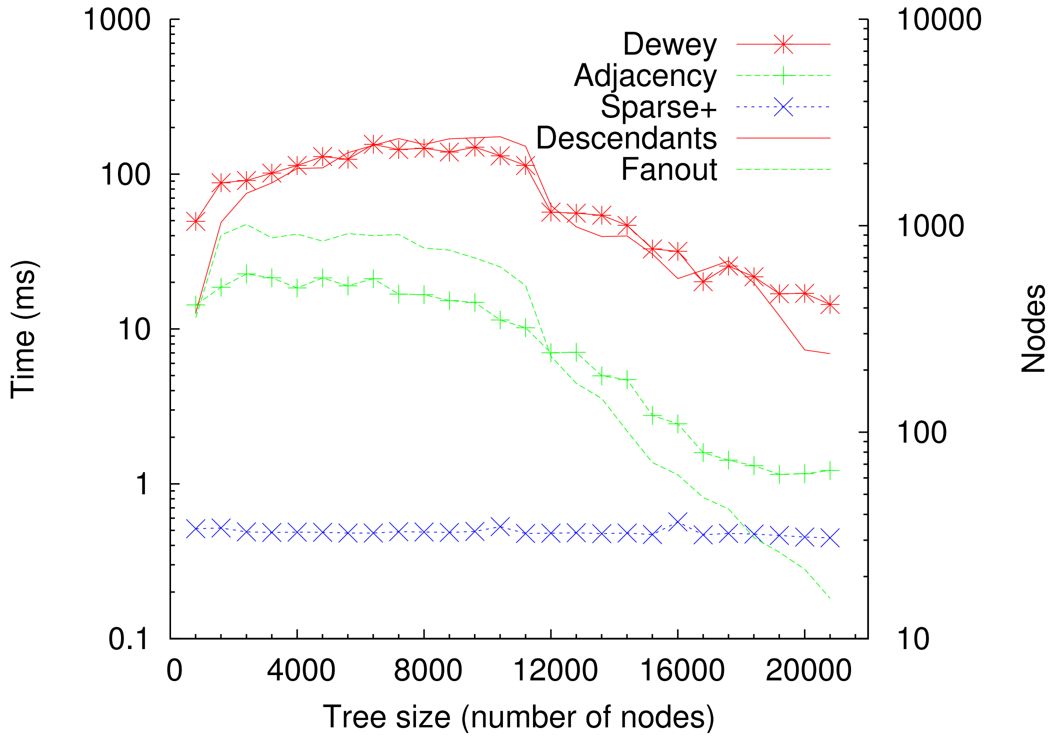


(Graph from [Dag08])

- Impact of the gap size on the number of hard updates:
  - "Sparse+": resets all gaps
  - "Sparse": shifts gaps

Outline

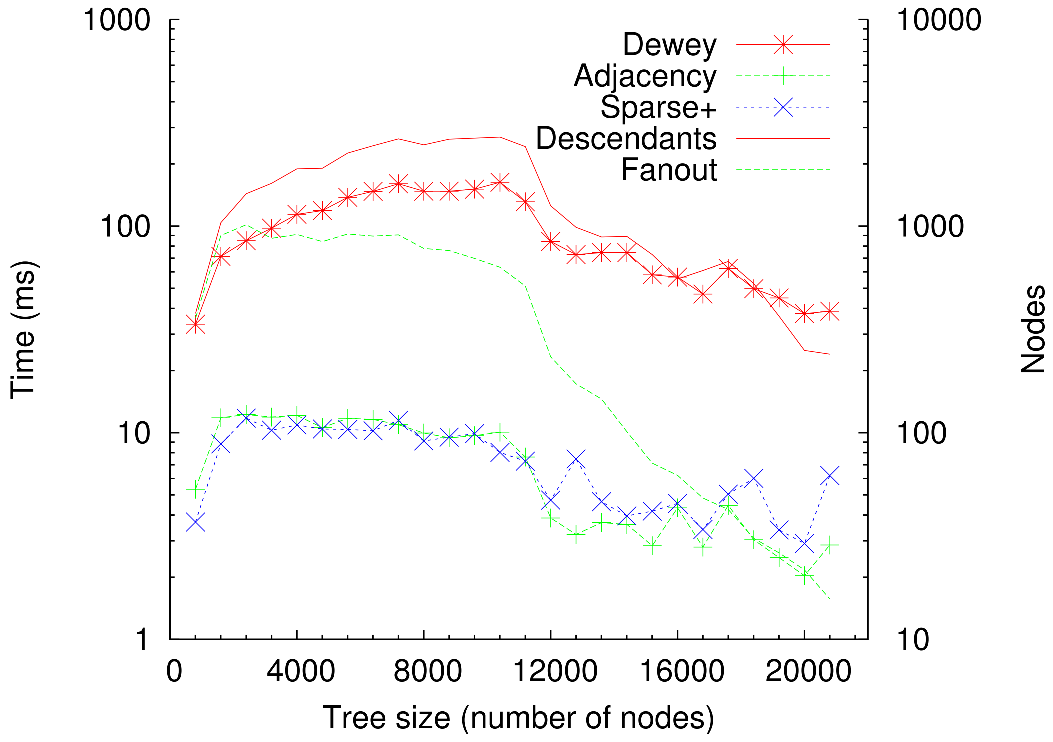
Delete Performance



(Graph from [Dag08])

- Delete performance of Adjacency List, Dewey, and Interval Encoding (Sparse+ [Dag08], gap size 100)
- Each data point in graph shows avg. runtime over 800 deletions
- Descendants: avg. number of descendants of deleted nodes
- Fanout: avg. fanout of deleted nodes

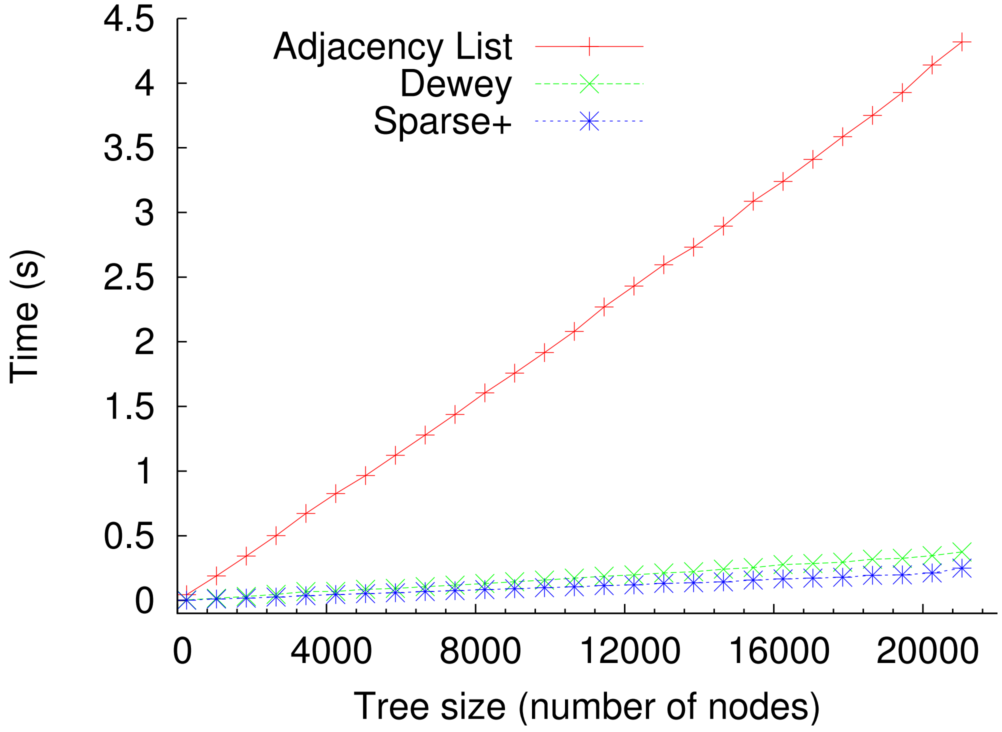
Insert Performance



(Graph from [Dag08])

- Insert performance of Adjacency List, Dewey, and Interval Encoding (Sparse+ [Dag08], gap size 100)
- Each data point in graph shows avg. runtime over 800 insertions
- Descendants: avg. number of descendants of inserted nodes
- Fanout: avg. fanout of inserted nodes

Efficiency of the Preorder Traversal



(Graph from [Dag08])

- Preorder traversal performance of Adjacency List, Dewey, and Interval Encoding (Sparse+ [Dag08], gap size 100)

Comparing the Encodings

	Adjacency	Dewey	Interval
+	• update very efficient • simple implementation	• preorder efficient • update efficiency: between others	• preorder very efficient • simple implementation
−	• preorder very inefficient	• update worst case is $O(n)$ • space overhead for storing paths	• insert is $O(n)$ on average (patch: sparse numbering)

Outline

Representing XML as a Tree

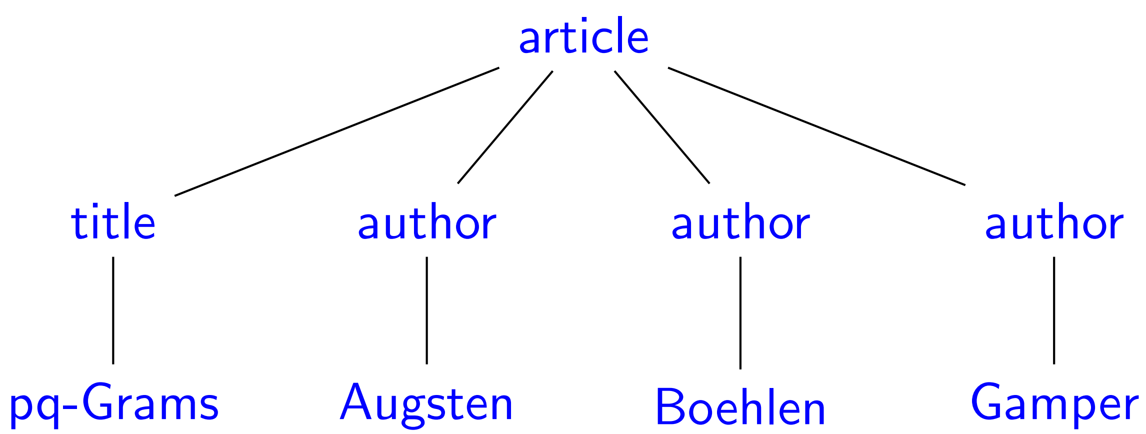
- Many possibilities – we will consider
  - single-label tree
  - double-label tree
- Pros/cons depend on application!

XML as a Single-Label Tree

- The XML document is stored as a tree with:
  - XML element: node labeled with element tag name
  - XML attribute: node labeled with attribute name
  - Text contained in elements/attributes: node labeled with the text-value
- Element nodes contain:
  - nodes of their sub-elements
  - nodes of their attributes
  - nodes with their text values
- Attribute nodes contain:
  - single node with their text value
- Text nodes are always leaves
- Order:
  - sub-element and text nodes are ordered
  - attributes are not ordered (approach: store them before all sub-elements, sort according to attribute name)

Example: XML as a Single-Label Tree

```
<article title='pq-Grams'>
  <author>Augsten</author>
  <author>Boehlen</author>
  <author>Gamper</author>
</article>
```

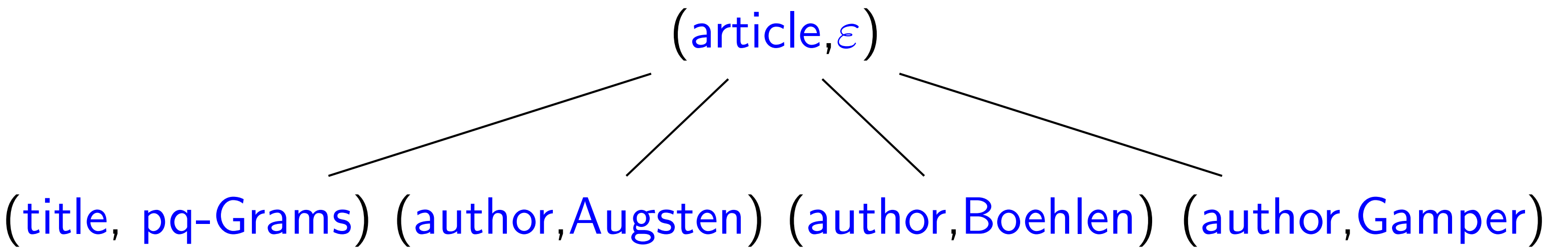


XML as a Double-Label Tree

- Node labels are pairs
- The XML document is stored as a tree with:
  - XML element: node labeled with (tag-name,text-value)
  - XML attribute: node labeled with (attribute-name,text-value)
- Element nodes contain:
  - nodes of their sub-elements and attributes
- Attribute nodes are always leaves
- Element nodes without attributes or sub-elements are leaves
- Order:
  - sub-element nodes are ordered
  - attributes are not ordered (approach: see previous slide)
- Limitation: Can represent
  - *either* elements with sub-elements and/or attributes
  - *or* elements with a text value

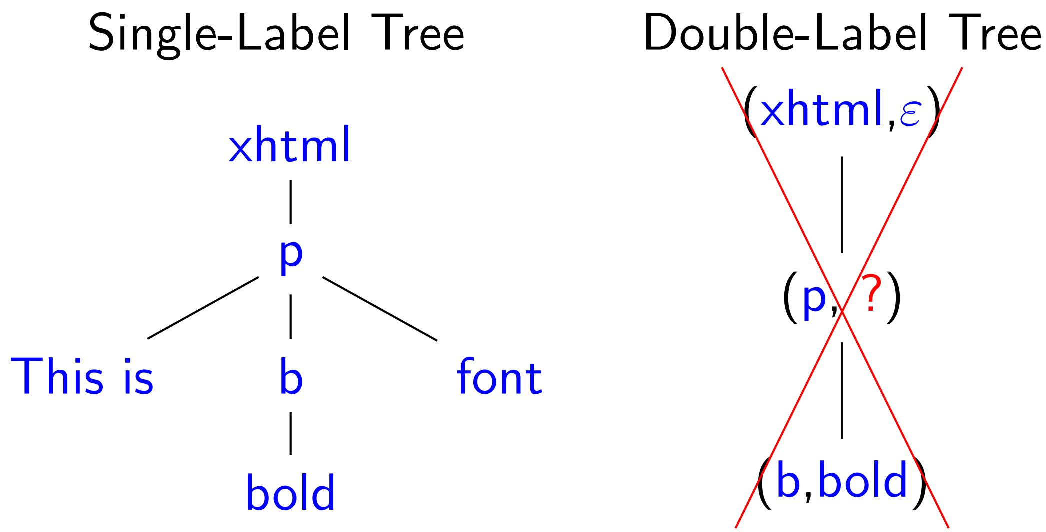
Example: XML as a Double-Label Tree

```
<article title='pq-Grams'>
  <author>Augsten</author>
  <author>Boehlen</author>
  <author>Gamper</author>
</article>
```



Example: Single- vs. Double-Label Tree

```
<xhtml>
  <p>This is <b>bold</b> font.</p>
<xhtml>
```



Parsing XML

We discuss two popular parsers for XML:

- DOM – Document Object Model
- SAX – Simple API for XML

DOM – Document Object Model

- W3C[2] standard for accessing and manipulating XML documents
- Tree-based: represents an XML document as a tree (single-label tree with additional node info, e.g. node type)
- Elements, attributes, and text values are nodes
- DOM parsers load XML into main memory
  - random access by traversing tree :-)
  - large XML documents do not fit into main memory :-(

[2] `http://www.w3schools.com/dom`

SAX – Simple API for XML

- "de facto" standard for parsing XML[3]
- Event-based: reports parsing events (e.g., start and end of elements)
  - no random access :-(
  - you see only one element/attribute at a time
  - you can parse (arbitrarily) large XML documents :-)
- Java API available for both, DOM and SAX
- For importing XML into a database: use SAX!

[3] `http://www.saxproject.org`

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