All Pairs

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Similarity functions

Function	Definition	eqo(r,s)	lb_r	ub_r
Jaccard	$\frac{\lvert r\cap s\rvert}{\lvert r\cup s\rvert}$	$\frac{t}{1+t}(\lvert r\rvert+\lvert s\rvert)$	$t\cdot\lvert r\rvert$	$\frac{\lvert r\rvert}{t}$
Cosine	$\frac{\lvert r\cap s\rvert}{\sqrt{\lvert r\rvert\cdot\lvert s\rvert}}$	$t\sqrt{\lvert r\rvert\cdot\lvert s\rvert}$	$t^2\lvert r\rvert$	$\frac{\lvert r\rvert}{t^2}$
Dice	$\frac{2\cdot\lvert r\cap s\rvert}{\lvert r\rvert+\lvert s\rvert}$	$\frac{t(\lvert r\rvert+\lvert s\rvert)}{2}$	$\frac{t\lvert r\rvert}{2-t}$	$\frac{(2-t)\lvert r\rvert}{t}$
Overlap	$\lvert r\cap s\rvert$	t	t	∞

Table: Similarity functions and set size bounds for set r and s.

All Pairs

Collection:

r_1 | 1 | 3 | 5 |

r_2 | 1 | 2 | 3 | 4 |

r_3 | 1 | 2 | 4 | 9 | 11 |

r_4 | 1 | 3 | 5 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

All Pairs

Collection:

r_1 | 1 | 3 | 5 |

r_2 | 1 | 2 | 3 | 4 |

r_3 | 1 | 2 | 4 | 9 | 11 |

r_4 | 1 | 3 | 5 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

- Jaccard: $J = \dfrac{|r \cap s|}{|r \cup s|}$
- $lb_r = t \cdot |r|$
- $\pi_r^i = |r| - \lceil \text{eqo}(r, r) \rceil + 1$
- $\text{eqo}(r, s) = \dfrac{t}{1+t} \cdot (|r| + |s|)$
- $\pi_r = |r| - \lceil lb_r \rceil + 1$

All Pairs

Threshold: $t = 0.5$

	$\lvert r_i \rvert$	$\text{eqo}(r, r)$	lb_{r_i}	π_{r_i}	$\pi^i_{r_i}$
r_1	3	2	1.5	2	2
r_2	4	$2.\dot{6}$	2	3	2
r_3	5	$3.\dot{3}$	2.5	3	2
r_4	10	$6.\dot{6}$	5	6	4

Algorithm

Algorithm 1: AllPairs(R, t)

input : R collection of sets, t similarity threshold
output: *res* set of result pairs (similarity at least t)

```
I = {};          /* I inverted list index covering prefix of sets */
foreach r in R do        /* process in ascending length order of r */
    M = {};     /* dictionary for candidate set. Key: candidate,
     value: number of intersecting tokens found so far. */
    for p ← 0 < π_r − 1 do   /* π_r: probing prefix length of r      */
        for s in I_{r[p]} do
            if |s| < lb_r then   /* lb_r: length bound               */
                remove index entry with s from I_{r[p]};
            else
                if s not in M then M[s] = 0;
                M[s] = M[s] + 1;
    for p ← 0 < π^i_r − 1 do   /* π^i_r: indexing prefix length of r  */
        I_{r[p]} = I_{r[p]} ∘ r;                  /* Add set r to index */
    /* Verify() verifies the candidates in M                          */
    res = res ∪ Verify(r, M, t);
```


Round 1

Current probing set: r_1 | 1 | 3 | 5 |

$lb_r = 1.5, \quad \pi_r = 2, \quad \pi^i_r = 2 \qquad M = \{\}$

Round 1

Current probing set: r_1 | 1 | 3 | 5 |

$lb_r = 1.5, \quad \pi_r = 2, \quad \pi^i_r = 2 \qquad M = \{\}$

$I = \{\}$

Round 1

Current probing set: r_1 | 1 | 3 | 5 |

$lb_r = 1.5, \quad \pi_r = 2, \quad \pi_r^i = 2 \qquad M = \{\}$

Index updated:

1	3
↓	↓
r_1	r_1

Round 1

Current probing set: r_1 | 1 | 3 | 5 |

$lb_r = 1.5, \quad \pi_r = 2, \quad \pi_r^i = 2 \qquad M = \{\}$

Index updated:

1	3
↓	↓
r_1	r_1

No verification

$res = \{\}$

Round 2

Current probing set: r_2 | 1 | 2 | 3 | 4 |

$lb_r = 2, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{\}$

Round 2

Current probing set: r_2 | 1 | 2 | 3 | 4 |

$lb_r = 2, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{\}$

Index:

1	3
↓	↓
r_1	r_1

Round 2

Current probing set: r_2 | 1 | 2 | 3 | 4 |

$lb_r = 2, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{(r_1, 2)\}$

Index:

1	3
↓	↓
r_1	r_1

Round 2

Current probing set: r_2 | 1 | 2 | 3 | 4 |

$lb_r = 2, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{(r_1, 2)\}$

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1	3
↓	↓
r_1	r_1

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Index updated:

1	2	3
↓	↓	↓
r_1	r_2	r_1
r_2		

Round 2

Current probing set: r_2 | 1 | 2 | 3 | 4 |

$lb_r = 2, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{(r_1, 2)\}$

Index updated:

1	2	3
↓	↓	↓
r_1	r_2	r_1
r_2		

Verification:

$|r_2 \cap r_1| \not\geq \lceil \text{eqo}(r_2, r_1) \rceil \iff (2 \not\geq 3)$

r_2 | 1 | 2 | 3 | 4 |

r_1 | 1 | 3 | 5 |

$res = \{\}$

Round 3

Current probing set: r_3 | 1 | 2 | 4 | 9 | 11 |

$lb_r = 2.5, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{\}$

Round 3

Current probing set: r_3 | 1 | 2 | 4 | 9 | 11 |

$lb_r = 2.5, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{\}$

Index:

1	2	3
↓	↓	↓
r_1	r_2	r_1
r_2		

Round 3

Current probing set: r_3 | 1 | 2 | 4 | 9 | 11 |

$lb_r = 2.5, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{(r_1, 1), (r_2, 2)\}$

Index:

1	2	3
↓	↓	↓
r_1	r_2	r_1
r_2		

Round 3

Current probing set: r_3 | 1 | 2 | 4 | 9 | 11 |

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Index:

1	2	3
↓	↓	↓
r_1	r_2	r_1
r_2		

Round 3

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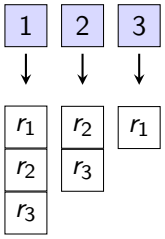
Index updated:



Round 3

Current probing set: r_3 | 1 | 2 | 4 | 9 | 11 |

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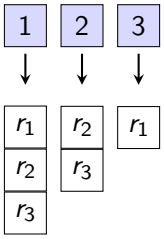
Index updated:



Verification 1:

$|r_3 \cap r_1| \not\geq \lceil \text{eqo}(r_3, r_1) \rceil \iff (1 \not\geq 3)$

r_3 | 1 | 2 | 4 | 9 | 11 |

r_1 | 1 | 3 | 5 |

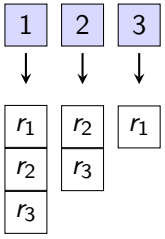
$res = \{\}$

Round 3

Current probing set: r_3 | 1 | 2 | 4 | 9 | 11 |

$lb_r = 2.5, \quad \pi_r = 3, \quad \pi_r^i = 2 \qquad M = \{(r_1, 1), (r_2, 2)\}$

Index updated:



Verification 2:

$|r_3 \cap r_2| \geq \lceil \text{eqo}(r_3, r_2) \rceil \iff (3 \geq 3)$

r_3 | 1 | 2 | 4 | 9 | 11 |

r_2 | 1 | 2 | 3 | 4 |

$res = \{(r_3, r_2)\}$

Round 4

Current probing set: r_4 | 1 | 3 | 5 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

$lb_r = 5, \quad \pi_r = 6, \quad \pi_r^i = 4 \qquad M = \{\}$

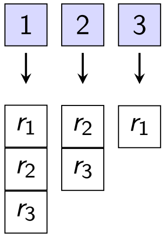
Round 4

Current probing set: r_4

1	3	5	8	9	10	11	12	13	14

$lb_r = 5, \quad \pi_r = 6, \quad \pi_r^i = 4 \qquad M = \{\}$

Index:



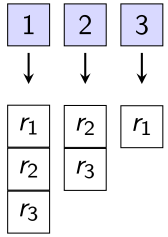
Round 4

Current probing set: r_4

1	3	5	8	9	10	11	12	13	14

$lb_r = 5, \quad \pi_r = 6, \quad \pi_r^i = 4 \qquad M = \{(r_3, 1)\}$

Index:



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Current probing set: r_4

1	3	5	8	9	10	11	12	13	14

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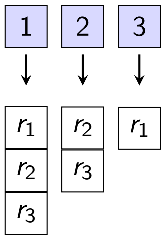
Index:



Round 4

Current probing set: r_4

1	3	5	8	9	10	11	12	13	14

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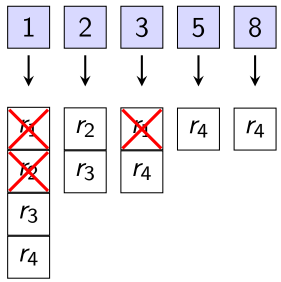
Index updated:



Round 4

Current probing set: r_4 | 1 | 3 | 5 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

$lb_r = 5, \quad \pi_r = 6, \quad \pi_r^i = 4 \qquad M = \{(r_3, 1)\}$

Index updated:

1	2	3	5	8
↓	↓	↓	↓	↓
~~r_1~~	r_2	~~r_1~~	r_4	r_4
~~r_2~~	r_3	r_4		
r_3				
r_4				

Verification:

$|r_4 \cap r_3| \not\geq \lceil \text{eqo}(r_4, r_3) \rceil \iff (2 \not\geq 5)$

r_4 | 1 | 3 | 5 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

r_3 | 1 | 2 | 4 | 9 | 11 |

$res = \{(r_3, r_2)\}$