Similarity Search

Pruning with Reference Sets

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Outline

1. Reference Sets
   - Definition
   - Upper and Lower Bound
   - Combined Approach

Outline

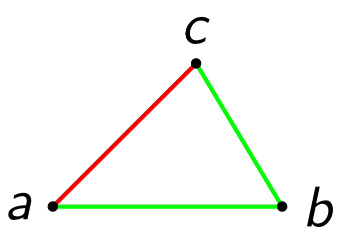
Reference Sets [?]

- Reference sets take advantage of the triangle inequality for metrics.
- An extended version of the triangle inequality is:

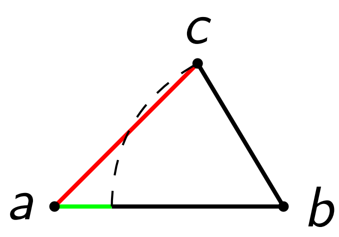
$$|\delta(a,b) - \delta(b,c)| \leq \delta(a,c) \leq \delta(a,b) + \delta(b,c),$$

where $\delta()$ is a metric distance function computed between a, b, and c.

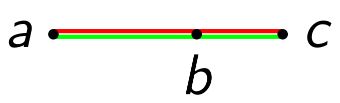
Illustration: Triangle Inequality



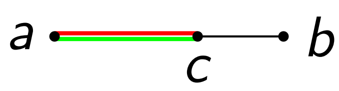
$\delta(a,c) < \delta(a,b) + \delta(b,c)$



$|\delta(a,b) - \delta(b,c)| < \delta(a,c)$



$\delta(a,c) = \delta(a,b) + \delta(b,c)$



$|\delta(a,b) - \delta(b,c)| = \delta(a,c)$

Notation

- S_1 and S_2 are sets of trees (unordered forests)
- $K \subseteq S_1 \cup S_2$ is called the reference set
- $\mathrm{T}_i \in S_1$ and $\mathrm{T}_j \in S_2$ are trees
- $k_l \in K$, $1 \leq l \leq |K|$, is a tree of the reference set

Reference Vector

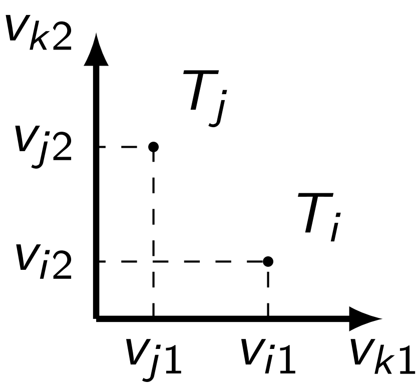
- v_i is a vector
  - of size $|K|$
  - that stores the distance from $\mathrm{T}_i \in S_1$ to each tree $k_l \in K$
  - the l-th element of v_i stores the edit distance between T_i and k_l:

$$v_{il} = \delta_t(\mathrm{T}_i, k_l)$$

- v_j is the respective vector for $\mathrm{T}_j \in S_2$

Metric Space

- Metric space of the reference set:
  - the elements of the reference set define the basis of a metric space
  - the vector v_k represents tree T_k as a point in this space
  - v_{kl} is the coordinate of the point on the l-th axis
- Example: Two trees T_i and T_j in the metric space defined by the reference set $K = \{k_1, k_2\}$.



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Upper and Lower Bound

- From the triangle inequality it follows that for all $1 \leq l \leq |K|$

$$|v_{il} - v_{jl}| \leq \delta_t(\mathrm{T}_i, \mathrm{T}_j) \leq v_{il} + v_{jl}$$

- Upper bound:

$$\delta_t(\mathrm{T}_i, \mathrm{T}_j) \leq \min_{l, 1 \leq l \leq |K|} v_{il} + v_{jl} = u_t(\mathrm{T}_i, \mathrm{T}_j)$$

- Lower bound:

$$\delta_t(\mathrm{T}_i, \mathrm{T}_j) \geq \max_{l, 1 \leq l \leq |K|} |v_{il} - v_{jl}| = l_t(\mathrm{T}_i, \mathrm{T}_j)$$

- Approximate join: match all trees with $\delta_t(\mathrm{T}_1, \mathrm{T}_2) \leq \tau$
  - Upper bound: If $u_t(\mathrm{T}_i, \mathrm{T}_j) \leq \tau$, then T_i and T_j match.
  - Lower bound: If $l_t(\mathrm{T}_i, \mathrm{T}_j) > \tau$, then T_i and T_j do *not* match.

Example: Similarity Join with Reference Sets

- $S_1 = \{T_1, T_2, T_3\}$, $S_2 = \{T_4, T_5, T_6\}$, $\tau = 2$
- Reference set $K = \{T_1\}$, $1 \leq l \leq |K| = 1$
- Distances:

	T_1	T_2	T_3	T_4	T_5	T_6
T_1	0	4	1	1	4	1
T_2		0	4	4	1	4
T_3			0	1	4	1
T_4				0	4	1
T_5					0	4
T_6						0

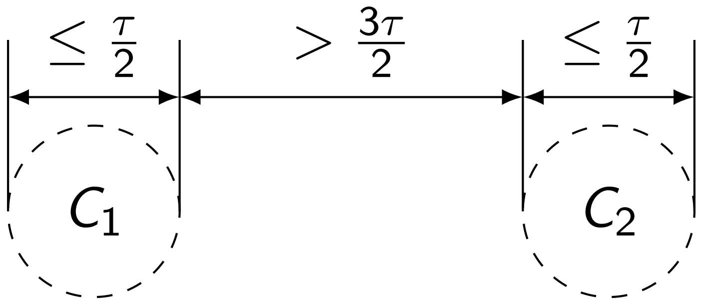
- Reference Vectors:
  $v_{1,1} = (0)$, $v_{2,1} = (4)$, $v_{3,1} = (1)$, $v_{4,1} = (1)$, $v_{5,1} = (4)$, $v_{6,1} = (1)$
- The clusters $C_1 = \{T_1, T_3, T_4, T_6\}$ and $C_2 = \{T_2, T_5\}$ are well separated.

Reference Set — Tradeoff

- Small reference set: efficient reference vector computation
  - we have to compute $|S|$ distances for each additional tree in the reference set to construct the vectors
  - for small reference sets the construction of the vectors is cheaper
- Large reference set: effective filters
  - large reference sets make u_t and l_t more effective
  - thus, once the reference vectors are computed, less distance computations are needed
- Where is the optimum?

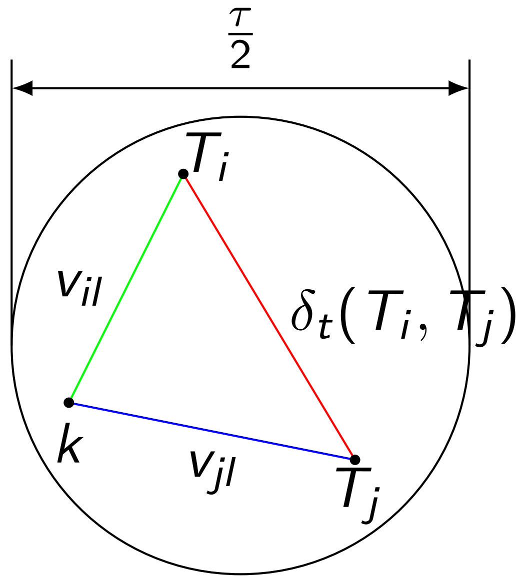
Well Separated Clusters

- We cluster the set $S = S_1 \cup S_2$.
- The clusters are well separated for threshold τ if
  - trees within a cluster have small distance (within $\frac{\tau}{2}$)
  - trees from different cluster have large distance (more than $\frac{3\tau}{2}$)
- Example: Two well separated clusters C_1 and C_2.



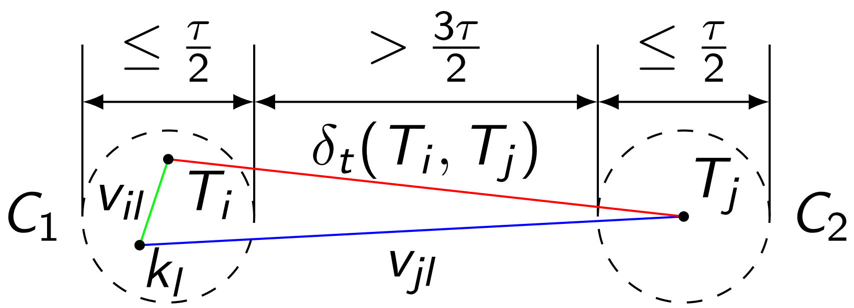
Trees within the Same Cluster — Upper Bound Applies

- Upper bound: $\delta_t(\mathrm{T}_i, \mathrm{T}_j) \le v_{il} + v_{jl} = \delta_t(\mathrm{T}_i, k_l) + \delta_t(k_l, \mathrm{T}_j)$
- Assumption: $\mathrm{T}_i \in S_1$ and $\mathrm{T}_j \in S_2$ are in the same cluster C.
  - The clusters are well separated.
  - The cluster C contains a reference tree $k_l \in K$.
- In this case $v_{il} \le \frac{\tau}{2}$ and $v_{jl} \le \frac{\tau}{2} \Rightarrow \delta_t(\mathrm{T}_i, \mathrm{T}_j) \le \tau$.
- Result: If T_i, T_j, and k_l are in the same cluster
  - from v_{il} and v_{jl} we conclude that T_1 and T_2 match
  - thus we need not to compute $\delta_t(\mathrm{T}_i, \mathrm{T}_j)$



Trees in Different Clusters — Lower Bound Applies

- Lower bound: $\delta_t(\mathrm{T}_i, \mathrm{T}_j) \geq |v_{il} - v_{jl}| = |\delta_t(\mathrm{T}_i, k_l) - \delta_t(k_l, \mathrm{T}_j)|$
- Assumption: $\mathrm{T}_i \in S_1$ is in cluster C_1, $\mathrm{T}_j \in S_2$ is in cluster C_2.
  - The clusters are well separated.
  - The cluster C_1 contains a reference tree $k_l \in K$.
- In this case $v_{il} \leq \frac{\tau}{2}$ and $v_{jl} > \frac{3\tau}{2} \Rightarrow \delta_t(\mathrm{T}_i, \mathrm{T}_j) > \tau$.
- Result: If T_i and k_l are in the same cluster, but T_j is in a different cluster
  - from v_{il} and v_{jl} we conclude that T_1 and T_2 do *not* match
  - thus we need not to compute $\delta_t(\mathrm{T}_i, \mathrm{T}_j)$



Optimum — Well Separated Clusters

- Optimum:
  - clusters are well separated
  - the reference set contains one tree per cluster
- Guha et al. [?] find clusters by sampling and estimate a reference set size.

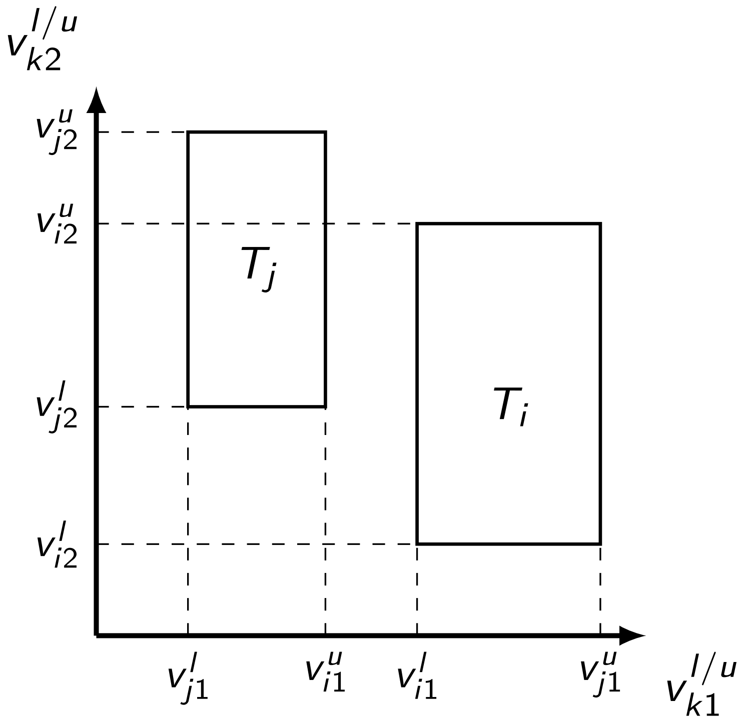
Outline

Combined Approach

- We combine the previous approaches:
  - lower bound with traversal strings
  - upper bound with constrained edit distance
  - reference sets
- Instead of one vector v_i we compute two vectors for each tree T_i:
  - lower bound: v_i^l contains the traversal string distance
  - upper bound: v_i^u contains the constrained edit distance

Metric Space

- Metric space of the reference set:
  - the elements of the reference set define the basis of a metric space
  - each tree T_k is represented by a rectangle in this metric space
  - v_k^l and v_k^u are two opposite corners of this rectangle
- Example: Two trees T_i and T_j in the metric space defined by the reference set $K = \{k_1, k_2\}$.



Combined Approach: Triangle Inequality

- The triangle equations changes as follows:
  - (a) For all $1 \leq l \leq |K|$

$$\delta_t(\mathrm{T}_i, \mathrm{T}_j) \quad \leq \quad v_{il}^u + v_{jl}^u$$

  - (b) For all $1 \leq l \leq |K|$

$$\delta_t(\mathrm{T}_i, \mathrm{T}_j) \geq \begin{cases} v_{jl}^l - v_{il}^u & \text{if } v_{jl}^l > v_{il}^u \\ v_{il}^l - v_{jl}^u & \text{if } v_{il}^l > v_{jl}^u \\ 0 & \text{otherwise} \end{cases}$$

- Note:
  - If $v_{jl}^l > v_{il}^u$ or $v_{il}^l > v_{jl}^u$ then $[v_{il}^l, v_{il}^u]$ and $[v_{jl}^l, v_{jl}^u]$ are disjoint intervals.
  - In all other cases we can not give a lower bound (other than 0).

Combined Approach: Upper and Lower Bounds

- Upper bound:

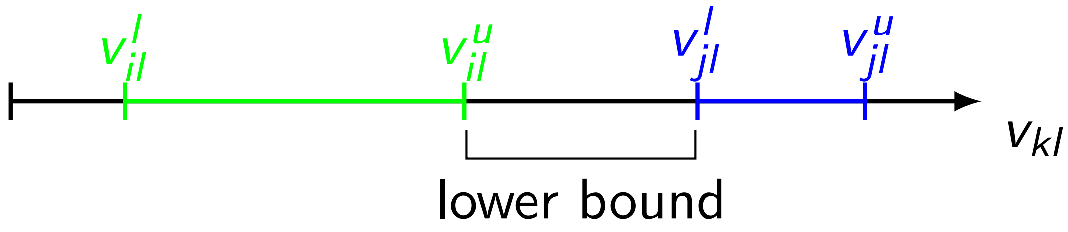
$$u_t(\mathrm{T}_i, \mathrm{T}_j) = \min_{l, 1 \leq l \leq |K|} v_{il}^u + v_{jl}^u$$

- Lower bound:

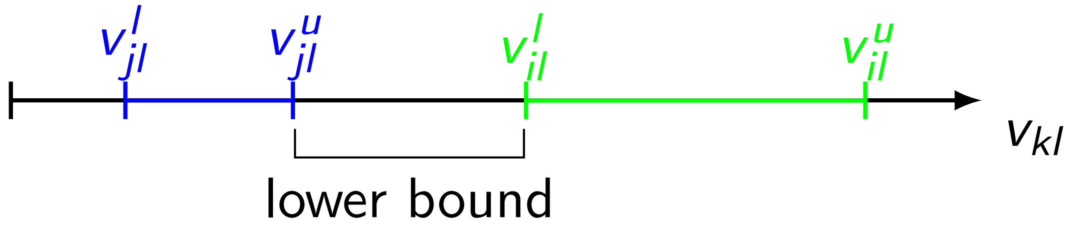
$$l_t(\mathrm{T}_i, \mathrm{T}_j) = \max_{l, 1 \leq l \leq |K|} \begin{cases} v_{jl}^l - v_{il}^u & \text{if } v_{jl}^l > v_{il}^u \\ v_{il}^l - v_{jl}^u & \text{if } v_{il}^l > v_{jl}^u \\ 0 & \text{otherwise} \end{cases}$$

Illustration: Cases for the Lower Bound

- Case $v_{jl}^{l} > v_{il}^{u}$: lower bound is $v_{jl}^{l} - v_{il}^{u}$



- Case $v_{il}^{l} > v_{jl}^{u}$: lower bound is $v_{il}^{l} - v_{jl}^{u}$



- All other cases ($[v_{il}^{l}, v_{il}^{u}]$ and $[v_{jl}^{l}, v_{jl}^{u}]$ overlap): no lower bound