

Similarity Search

The Binary Branch Distance

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Outline

- 1 Binary Branch Distance
 - Binary Representation of a Tree
 - Binary Branches
 - Lower Bound for the Edit Distance
 - Complexity
- 2 Conclusion

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Binary Tree

- In a **binary tree**
 - each node has **at most two children**;
 - **left child** and **right child** are distinguished:
a node can have a right child without having a left child;
- **Notation:** $T_B = (N, E_l, E_r)$
 - T_B denotes a binary tree
 - N are the nodes of the binary tree
 - E_l and E_r are the edges to the left and right children, respectively
- **Full binary tree:**
 - binary tree
 - each node has exactly zero or two children.

Example: Binary Tree

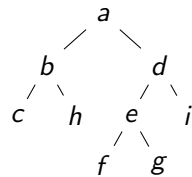
- Two different binary trees: $T_B = (N, E_l, E_r)$

$$T_{B1} = (\{a, b, c, d, e, f, g\}, \{(a, b), (b, c), (d, e), (e, f)\}, \{(a, d), (e, g)\})$$

$$T_{B2} = (\{a, b, c, d, e, f, g\}, \{(a, b), (b, c), (e, f)\}, \{(a, d), (d, e), (e, g)\})$$



- A full binary tree:

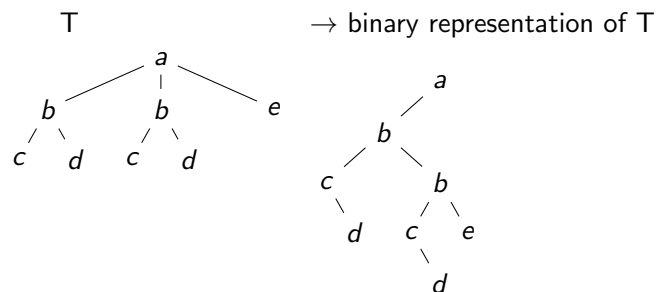


Binary Representation of a Tree

- Binary tree transformation:
 - link all neighboring siblings in a tree with edges
 - delete all parent-child edges except the edge to the first child
- Transformation maintains
 - label information
 - structure information
- Original tree can be reconstructed from the binary tree:
 - a left edge represents a parent-child relationship in the original tree
 - a right edge represents a right-sibling relationship in the original tree

Example: Binary Tree Transformation

- Represent tree T as a binary tree:

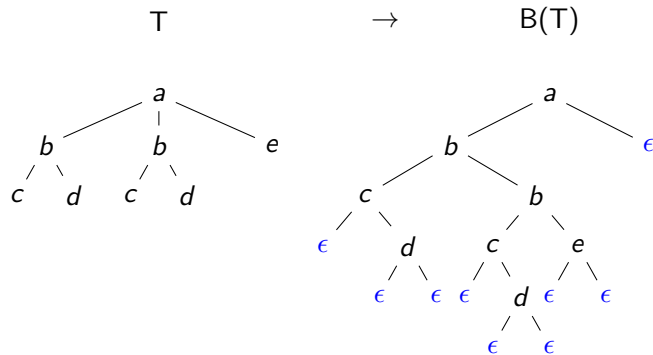


Normalized Binary Tree Representation

- We extend the binary tree with null nodes ϵ as follows:
 - a null node for each missing left child of a non-null node
 - a null node for each missing right child of a non-null node
- Note: Leaf nodes get two null-children.
- The resulting normalized binary representation
 - is a full binary tree
 - all non-null nodes have two children
 - all leaves are null nodes (and all null nodes are leaves)

Example: Normalized Binary Tree

- Transforming T to the normalized binary tree $B(T)$:



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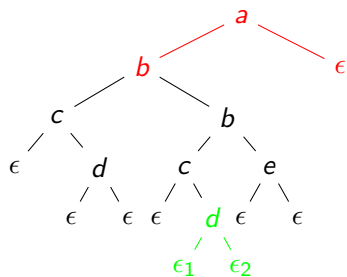
Binary Branch

- A binary branch $BiB(v)$ is
 - a subtree of the normalized binary tree $B(T)$
 - consisting of a non-null node v and its two children

- Example:

$$BiB(a) = (\{a, b, \epsilon\}, \{(a, b)\}, \{(a, \epsilon)\})$$

$$BiB(d) = (\{d, \epsilon_1, \epsilon_2\}, \{(d, \epsilon_1)\}, \{(d, \epsilon_2)\})^1$$

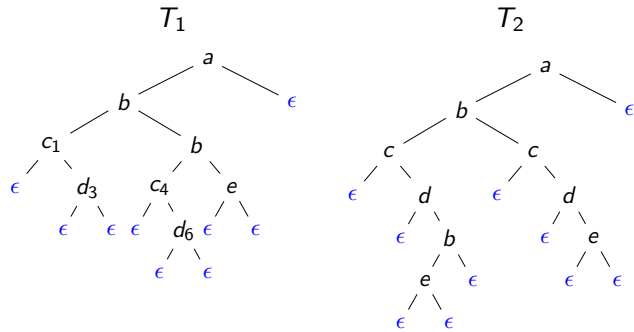


¹Although the two null nodes have identical labels (ϵ), they are different nodes. We emphasize this by showing their IDs in subscript.

Binary Branches of Trees and Datasets

- Binary branches can be serialized as strings:
 - $BiB(v) = (\{v, a, b\}, \{(v, a)\}, \{(v, b)\}) \rightarrow \lambda(v) \circ \lambda(a) \circ \lambda(b)$
 - we can sort these strings ($\epsilon > \lambda(v)$ for all non-null nodes v)
- Binary branch sets:
 - $BiB(T)$ is the set of all binary branches of $B(T)$
 - $BiB(S) = \bigcup_{T \in S} BiB(T)$ is the set of all binary branches of dataset S
 - $BiB_{sort}(S)$ is the vector of sorted serialized strings of $BiB(S)$
- Note:
 - nodes are unique in the tree, thus binary branches are unique
 - labels are *not* unique, thus the serialized binary branches are *not* unique

Example: Binary Branches of Trees and Datasets

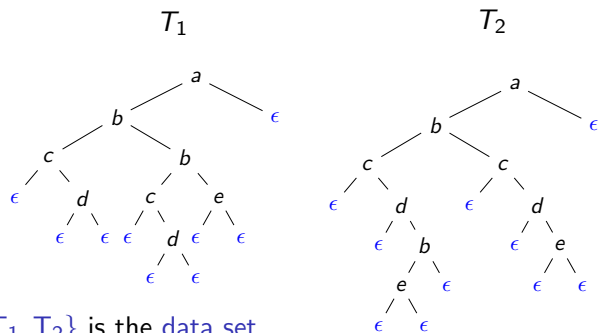


- $BiB(c_1) \neq BiB(c_4)$:
 - $BiB(c_1) = (\{c_1, \epsilon_2, d_3\}, \{(c_1, \epsilon_2)\}, \{(c_1, d_3)\})$
 - $BiB(c_4) = (\{c_4, \epsilon_5, d_6\}, \{(c_4, \epsilon_5)\}, \{(c_4, d_6)\})$
- Serialization of both, $BiB(c_1)$ and $BiB(c_2)$, is identical: 'c ϵ d'
- Sorted vector of serialized strings of $BiB(S)$, where $S = \{T_1, T_2\}$:
 $BiB_{sort}(S) = (abc\epsilon, bcb, bcc, bce, be\epsilon, c\epsilon d, d\epsilon b, d\epsilon e, d\epsilon\epsilon, e\epsilon\epsilon)$

Binary Branch Vector

- The **binary branch vector** $BBV(T)$
 - is a representation of the binary branch set $BiB(T)$
- **Construction** of the binary branch vector $BBV(T)$:
 - compute $BiB_{sort}(S)$ (*serialize and sort* $BiB(S)$)
 - b_i is the i -th serialized binary branch in sort order ($b_i = BiB_{sort}(S)[i]$)
 - $BBV(T)[i]$ is the number of binary branches in $B(T)$ that serialize to b_i
- **Note:** $BBV(T)[i]$ is zero if b_i does not appear in $BiB(T)$

Example: Binary Branch Vectors



- $S = \{T_1, T_2\}$ is the **data set**
- $BiB_{sort}(S)$ is the vector of sorted **serialized strings** of $BiB(S)$
- $BBV(T_j)$ is the **binary branch vector** of T_j
- the vector of serialized strings and the binary branch vectors are:

$BiB_{sort}(S)$	$abc\epsilon$	bcb	bcc	bce	$be\epsilon$	$c\epsilon d$	$d\epsilon b$	$d\epsilon e$	$d\epsilon\epsilon$	$e\epsilon\epsilon$
$BBV(T_1)$	1	1	0	1	0	2	0	0	2	1
$BBV(T_2)$	1	0	1	0	1	2	1	1	0	2

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Binary Branch Distance [YKT05]

Definition (Binary Branch Distance)

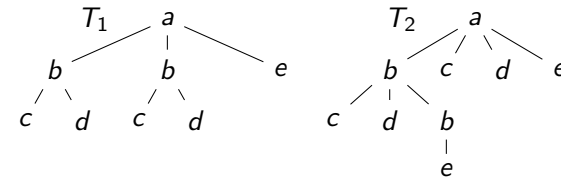
Let $BBV(T) = (b_1, \dots, b_k)$ and $BBV(T') = (b'_1, \dots, b'_k)$ be binary branch vectors of trees T and T' , respectively. The binary branch distance of T and T' is

$$\delta_B(T, T') = \sum_{i=1}^k |b_i - b'_i|.$$

- **Intuition:** We count the binary branches that do not match between the two trees.

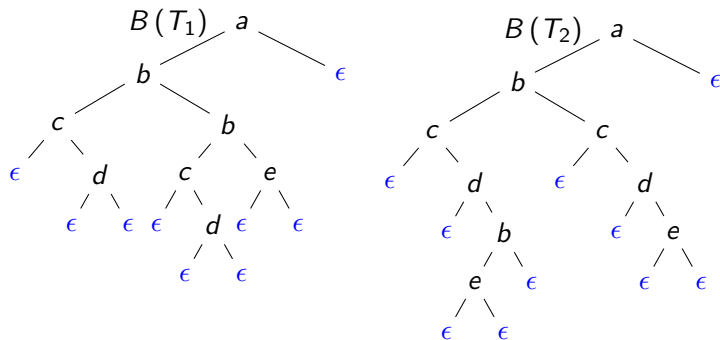
Example: Binary Branch Distance

- We compute the binary branch distance between T_1 and T_2 :



Example: Binary Branch Distance

- The normalized binary tree representations are:



Example: Binary Branch Distance

- The binary branch vectors of T_1 and T_2 are:

$BiB_{sort}(S)$	$ab\epsilon$	$bc\epsilon$	bcc	bce	$be\epsilon$	$cc\epsilon$	$d\epsilon b$	$d\epsilon\epsilon$	$d\epsilon\epsilon$	$\epsilon\epsilon\epsilon$
$BBV(T_1)$	1	1	0	1	0	2	0	0	2	1
$BBV(T_2)$	1	0	1	0	1	2	1	1	0	2

- The binary branch distance is

$$\begin{aligned} \delta_B(T_1, T_2) &= \sum_{i=1}^{10} |b_{1,i} - b_{2,i}| \\ &= |1 - 1| + |1 - 0| + |0 - 1| + |1 - 0| + |0 - 1| + \\ &\quad |2 - 2| + |0 - 1| + |0 - 1| + |2 - 0| + |1 - 2| \\ &= 9, \end{aligned}$$

where $b_{1,i}$ and $b_{2,i}$ are the i -th dimension of the vectors $BBV(T_1)$ and $BBV(T_2)$, respectively.

Lower Bound Theorem

Theorem (Lower Bound)

Let T and T' be two trees. If the tree edit distance between T and T' is $\delta_t(T, T')$, then the binary branch distance between them satisfies

$$\delta_B(T, T') \leq 5 \times \delta_t(T, T').$$

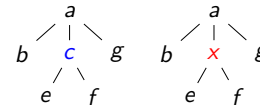
Proof (Sketch — Full Proof in [YKT05]).

- Each node v appears in at most two binary branches.
- *Rename*: Renaming a node causes at most two binary branches in each tree to mismatch. The sum is 4.
- Similar rationale for *insert* and its complementary operation *delete* (at most 5 binary branches mismatch).

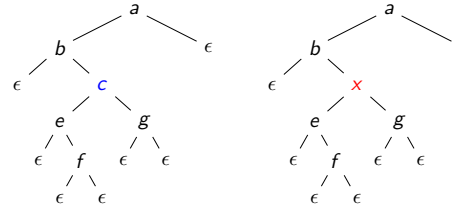
□

Proof Sketch: Illustration for Rename

- transform T_1 to T_2 : $ren(c, x)$



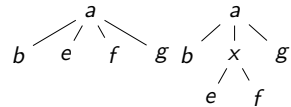
- binary trees $B(T_1)$ and $B(T_2)$



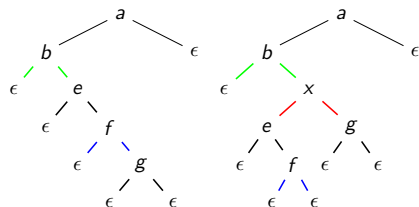
- Two binary branches $(b\epsilon c, ceg)$ exist only in $B(T_1)$
- Two binary branches $(b\epsilon x, xeg)$ exist only in $B(T_2)$
- $\delta_t(T_1, T_2) = 1$ (1 rename)
- $\delta_B(T_1, T_2) = 4$ (4 binary branches different)

Proof Sketch: Illustration for Insert

- transform T_1 to T_2 : $ins(x, a, 2, 2)$



- binary trees $B(T_1)$ and $B(T_2)$



- Two binary branches $(b\epsilon e, feg)$ exist only in $B(T_1)$
- Three binary branches $(b\epsilon x, f\epsilon e, xeg)$ exist only in $B(T_2)$
- $\delta_t(T_1, T_2) = 1$ (1 insertion)
- $\delta_B(T_1, T_2) = 5$ (5 binary branches different)

Proof Sketch

- In general it can be shown that
 - *Rename* changes at most 4 binary branches
 - *Insert* changes at most 5 binary branches
 - *Delete* changes at most 5 binary branches
- Each edit operation changes at most 5 binary branches, thus

$$\delta_B(T, T') \leq 5 \times \delta_t(T, T').$$

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Complexity: Binary Branch Distance

- Compute the distance between two trees of size $O(n)$:
($S = \{T_1, T_2\}$, $n = \max\{|T_1|, |T_2|\}$)
- Construction of the **binary branch vectors** $BBV(T_1)$ and $BBV(T_2)$:
 1. $BiB(S)$ – compute the binary branches of T_1 and T_2 :
 $O(n)$ time and space (traverse T_1 and T_2)
 2. $BiB_{sort}(S)$ – sort *serialized* binary branches of $BiB(S)$:
 $O(n \log n)$ time and $O(n)$ space
 3. construct $BBV(T_1)$ and $BBV(T_2)$:
 - (a) traverse all binary branches: $O(n)$ time and space
 - (b) for each binary branch find position i in $BiB_{sort}(S)$: $O(n \log n)$ time
(binary search in $BiB_{sort}(S)$ for n binary branches)
 - (c) $BBV(T)[i]$ is incremented: $O(1)$
- Computing the **distance**:
 - the two binary branch vectors are of size $O(n)$
 - computing the distance has time complexity $O(n)$
(subtracting two binary branch vectors)
- The **overall complexity** is $O(n \log n)$ time and $O(n)$ space.

Improving the Time Complexity with a Hash Function

- **Note:** Improvement using a hash function:
 - we assume a hash function that maps the $O(n)$ binary branches to $O(n)$ buckets without collision
 - we do *not* sort $BiB(S)$
 - position i in the vector $BBV(T)$ is computed using the hash function
 - $O(n)$ time (instead of $O(n \log n)$) and $O(n)$ space
- In the following we assume the sort algorithm with $O(n \log n)$ runtime.

Complexity for Similarity Joins

- Join two sets with N trees each (tree size: n):
- **Compute Binary Branch Vectors (BBVs)**:
 $O(Nn \log(Nn))$ time, $O(N^2n)$ space
 - BBVs are of size $O(Nn)$
 - time: sort $O(Nn)$ binary branches / $O(Nn)$ binary searches in BBVs
 - space: $O(N)$ BBVs must be stored
- **Compute Distances**: $O(N^3n)$ time
 - computing the distance between two trees has $O(Nn)$ time complexity
(subtracting two binary branch vectors)
 - $O(N^2)$ distance computations required
- **Overall Complexity**: $O(N^3n + Nn \log n)$ [†] time and $O(N^2n)$ space

[†] $O(N^3n + Nn \log(Nn)) = O(N^3n + Nn \log N + Nn \log n) = O(N^3n + Nn \log n)$

Summary

- Binary Branch Distance
 - lower bound of the **unit cost** tree edit distance
 - trees are split into binary branches (small subgraphs)
 - similar trees have many common binary branches
 - complexity $O(n \log n)$ time



Rui Yang, Panos Kalnis, and Anthony K. H. Tung.

Similarity evaluation on tree-structured data.

In *Proceedings of the ACM SIGMOD International Conference on Management of Data*, pages 754–765, Baltimore, Maryland, USA, June 2005. ACM Press.