

# Similarity Search

## The $pq$ -Gram Distance

Nikolaus Augsten

nikolaus.augsten@sbg.ac.at  
Department of Computer Sciences  
University of Salzburg



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# Outline

- 1 The  $pq$ -Gram Distance
  - Definition
  - Algorithm
  - Fanout Weighting and Lower Bound
  - Experiments
- 2 Conclusion

# Outline

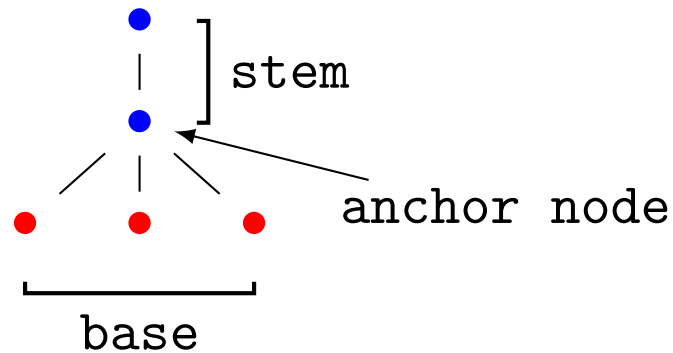
- 1 The  $pq$ -Gram Distance
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# $pq$ -Grams – Intuition

- $q$ -Grams for strings:
  - split string into substrings ( $q$ -grams) of length  $q$
  - strings with many common substrings are similar
- $pq$ -Grams for trees:
  - split tree into small subtrees ( $pq$ -grams) of the same shape
  - trees with many common subtrees are similar

# $pq$ -Grams

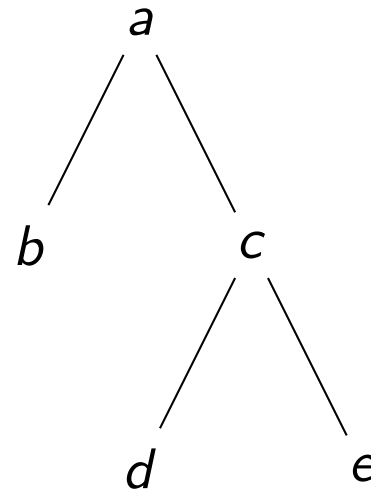
- The shape of a  $pq$ -gram ( $p=2$ ,  $q=3$ ):



- $p$  nodes (anchor node and  $p-1$  ancestors) form the **stem**
- $q$  nodes ( $q$  consecutive children of the anchor node) form the **base**

# $pq$ -Extended Tree

- **Problem:** How can we split the following tree  $T$  into 2, 3-grams?

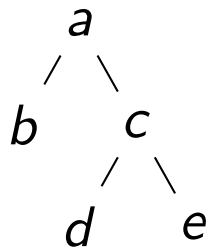


- **Solution:** Extend tree  $T$  with dummy nodes (●):
  - $p-1$  ancestors to the root node
  - $q-1$  children before the first and after the last child of each non-leaf
  - $q$  children for each leaf
- The result is the  $pq$ -extended tree  $T^{pq}$ .

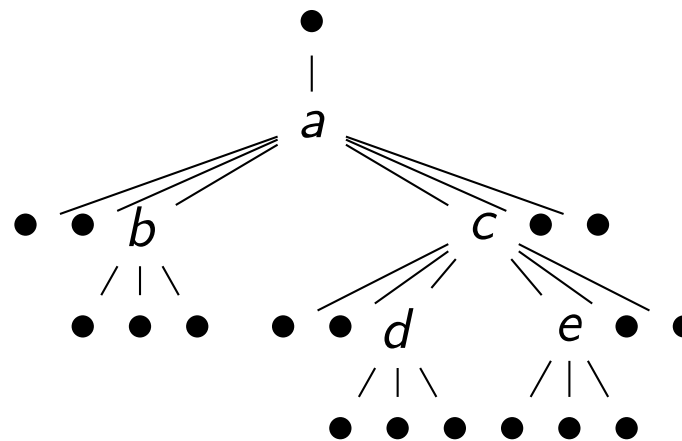
# Example: Extended Tree

- An example tree  $T$  and its extended tree  $T^{pq}$  ( $p=2, q=3$ ):

$T$



2, 3-extended tree  $T^{2,3}$



## Definition: $pq$ -Gram [ABG05]

### Definition ( $pq$ -Gram)

Let  $T$  be a tree,  $T^{p,q}$  the respective extended tree,  $p > 0$ ,  $q > 0$ . A subtree of  $T^{p,q}$  is a  *$pq$ -gram*  $g$  of  $T$  iff

- (a)  $g$  has  $q$  leaf nodes and  $p$  non-leaf nodes,
- (b) all leaf nodes of  $g$  are children of a single node  $a \in N(g)$  with fanout  $q$ , called the *anchor node*,
- (c) the leaf nodes of  $g$  are consecutive siblings in  $T^{p,q}$ .

- **Stem:** anchor node and its ancestors in the  $pq$ -gram.
- **Base:** children of the anchor node in the  $pq$ -gram.

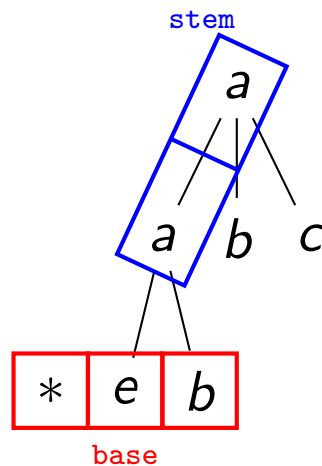
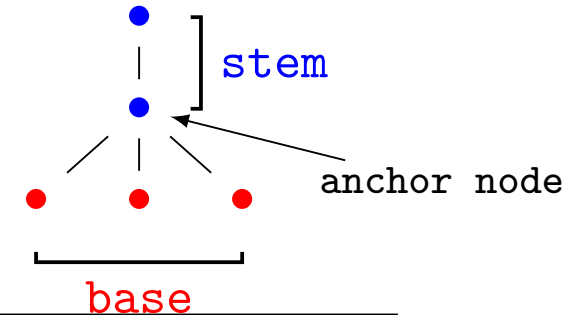
### Definition ( $pq$ -Gram Profile)

The  $pq$ -gram profile,  $P_T$ , of a tree  $T$  is the set of all its  $pq$ -grams.

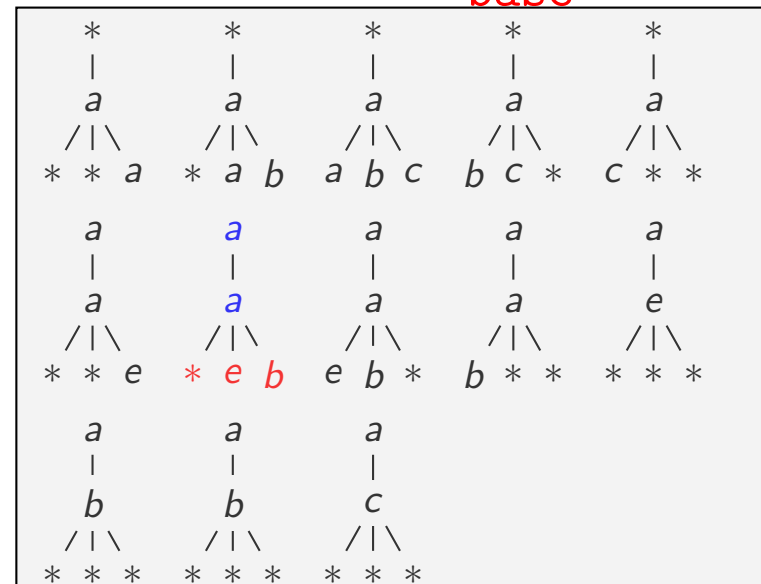


# Example: Systematically Split Tree

- $pq$ -Gram: small subtree with stem and base  
Example:  $p = 2, q = 3$
- Systematically split tree into  $pq$ -grams
- $pq$ -Gram profile: set of all  $pq$ -grams of a tree.

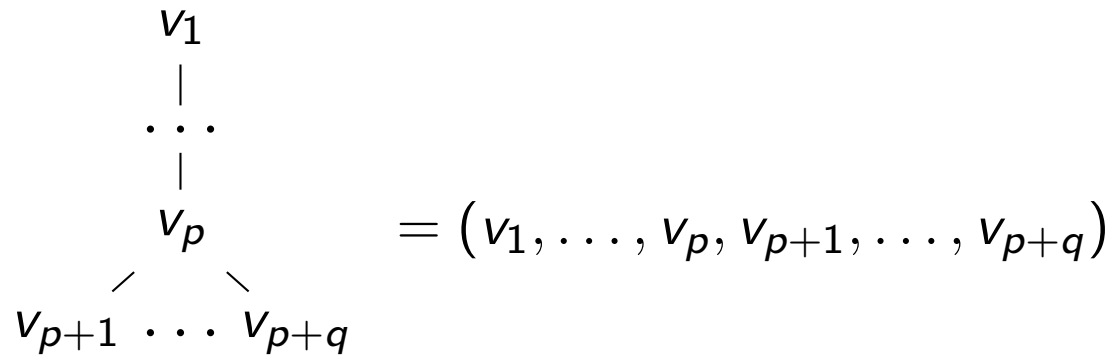


$P(T)$



# Label Tuples

- **Linear encoding** of a  $pq$ -gram  $g$  with anchor node  $v_p$ :  
(traverse  $pq$ -gram in preorder)



- **Label tuple**: tuple of the  $pq$ -gram's node labels

$$\lambda(g) = (\lambda(v_1), \dots, \lambda(v_{p+q}))$$

for the  $pq$ -gram  $g = (v_1, \dots, v_{p+q})$ .

# $pq$ -Gram Index

## Definition ( $pq$ -Gram Index)

Let  $T$  be a tree with profile  $P_T$ ,  $p > 0$ ,  $q > 0$ . The  $pq$ -gram index,  $\mathcal{I}$ , of tree  $T$  is the **bag of all label tuples** of  $T$ ,

$$\mathcal{I}(T) = \bigcup_{g \in P_T} \lambda(g)$$

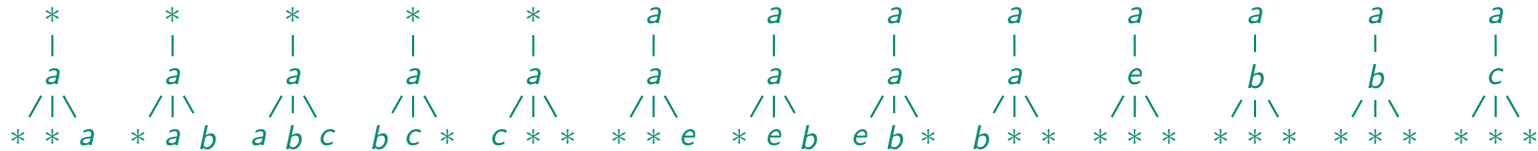
- Note:
  - $pq$ -grams are unique within a tree
  - but: different  $pq$ -grams may yield identical label tuples
  - thus the  $pq$ -gram index may contain duplicates

# Storing the $pq$ -Gram Index Efficiently

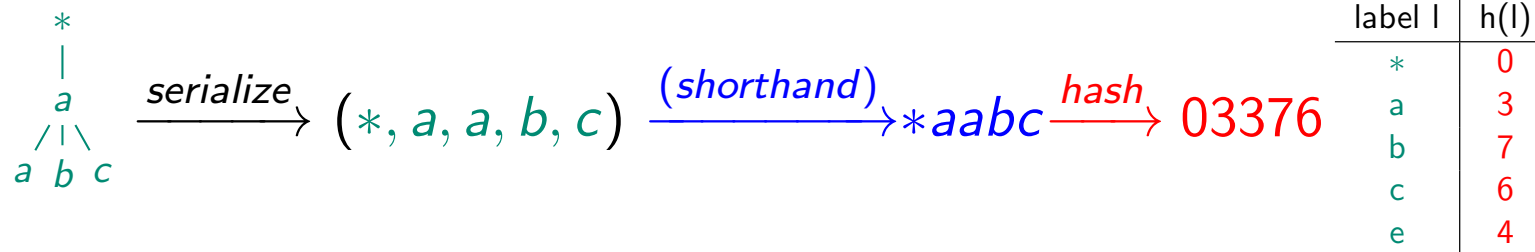
- **Problem:** How to store node labels efficiently?
  - **Long labels:** large storage overhead
  - **Varying label length:** in a relational database, the inefficient VARCHAR type must be used instead of the efficient CHAR type
- **Solution:** Hashing
  - compute fingerprint hash for labels
  - store concatenation of the hashed labels
- **Fingerprint hash function** (e.g., Karp-Rabin [KR87]):
  - maps a string  $s$  to a hash value  $h(s)$
  - $h(s)$  is of fixed length
  - $h(s)$  is unique with high probability  
(for two different strings  $s_1 \neq s_2$ ,  $h(s_1) \neq h(s_2)$  with high probability)

# Overview: The *pq*-Gram Index

- *pq*-Gram profile:



- Hashing: map *pq*-gram to integer:



Note: labels may be strings of arbitrary length!

- *pq*-Gram index: bag of hashed *pq*-grams

$$\mathcal{I}(T) = \{03003, 03037, 03376, 03760, 03600, 33004, 33047, 33470, 33700, 37000, 36000, 34000, 37000\}$$

**Intuition:** similar trees have similar *pq*-gram indexes.

# The $pq$ -Gram Distance

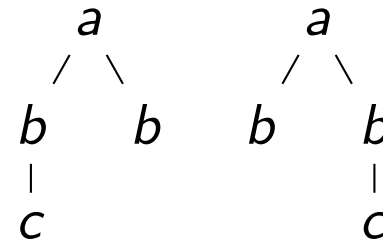
## Definition ( $pq$ -Gram Distance)

The  $pq$ -gram distance between two trees,  $T$  and  $T'$ , is defined as

$$\delta_g(T, T') = |\mathcal{I}(T) \uplus \mathcal{I}(T')| - 2|\mathcal{I}(T) \cap \mathcal{I}(T')|$$

- Metric normalization to  $[0..1]$ :  $\delta'_g(T, T') = \frac{\delta_g(T, T')}{|\mathcal{I}(T) \uplus \mathcal{I}(T')| - |\mathcal{I}(T) \cap \mathcal{I}(T')|}$
- Pseudo-metric properties hold for normalization [ABG10]:
  - ✓ self-identity:  $x = y \not\Rightarrow \delta_g(x, y) = 0$
  - ✓ symmetry:  $\delta_g(x, y) = \delta_g(y, x)$
  - ✓ triangle inequality:  $\delta_g(x, z) \leq \delta_g(x, y) + \delta_g(y, z)$

- Different trees may have identical indexes:



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## Main Memory Algorithm (I)

```
CREATEINDEX( $T, r, \mathcal{I}, stem, p, q$ )
```

```
 $stem := \text{shift}(stem, \lambda(r))$ 
```

```
 $base$ : shift register of size  $q$  (filled with  $*$ )
```

```
if  $r$  is a leaf then
```

```
   $\mathcal{I} := \mathcal{I} \cup \{stem \circ base\}$ 
```

```
else
```

```
  for each child  $c$  (from left to right) of  $r$  do
```

```
     $base := \text{shift}(base, \lambda(c))$ 
```

```
     $\mathcal{I} := \mathcal{I} \cup \{stem \circ base\}$ 
```

```
     $\mathcal{I} := \text{CREATEINDEX}(T, c, \mathcal{I}, stem, p, q)$ 
```

```
  for  $k := 1$  to  $q - 1$ 
```

```
     $base := \text{shift}(base, *)$ 
```

```
     $\mathcal{I} := \mathcal{I} \cup \{stem \circ base\}$ 
```

```
return  $\mathcal{I}$ 
```



# Main Memory Algorithm (II)

- **Input** of  $\text{CREATEINDEX}(T, r, \mathcal{I}, \text{stem}, p, q)$ :
  - a subtree of  $T$  rooted in  $r$
  - the  $pq$ -gram index  $\mathcal{I}$  computed so far
  - the stem  $\text{stem}$  of  $r$ 's parent
  - the parameters  $p$  and  $q$
- **Output** of  $\text{CREATEINDEX}(T, r, \mathcal{I}, \text{stem}, p, q)$ :  
 $pq$ -gram index including
  - the input index  $\mathcal{I}$
  - the  $pq$ -gram index of  $r$  and all its descendants  
i.e., the  $pq$ -grams (label tuples) with anchor node  $r$  or a descendant of  $r$

# Main Memory Algorithm (III)

- $pq$ -GRAM-INDEX( $T, p, q$ ) computes the  $pq$ -gram index for a complete tree  $T$ :

$pq$ -GRAM-INDEX( $T, p, q$ )

*stem*: shift register of size  $p$  (filled with  $*$ )

$\mathcal{I}$ : empty index

$\mathcal{I} = \text{CREATEINDEX}(T, \text{root}(T), \mathcal{I}, \textit{stem}, p, q)$

**return**  $\mathcal{I}$

# Complexity of the $pq$ -Gram Index Algorithm

## Theorem ( $pq$ -Gram Index Complexity)

The  $pq$ -gram index of a tree  $T$  with size  $|T|$  can be computed in  $O(|T|)$  time.

## Proof.

Each recursive call of *createIndex()* processes one node in constant time, and each node is processed exactly once. □

# Size of the $pq$ -Gram Index

## Theorem (Size of the $pq$ -Gram Index)

Let  $T$  be a tree with  $l$  leaves and  $i$  non-leaves. The size of the  $pq$ -gram index of  $T$  is

$$|\mathcal{I}^{pq}(T)| = 2l + qi - 1.$$

## Proof.

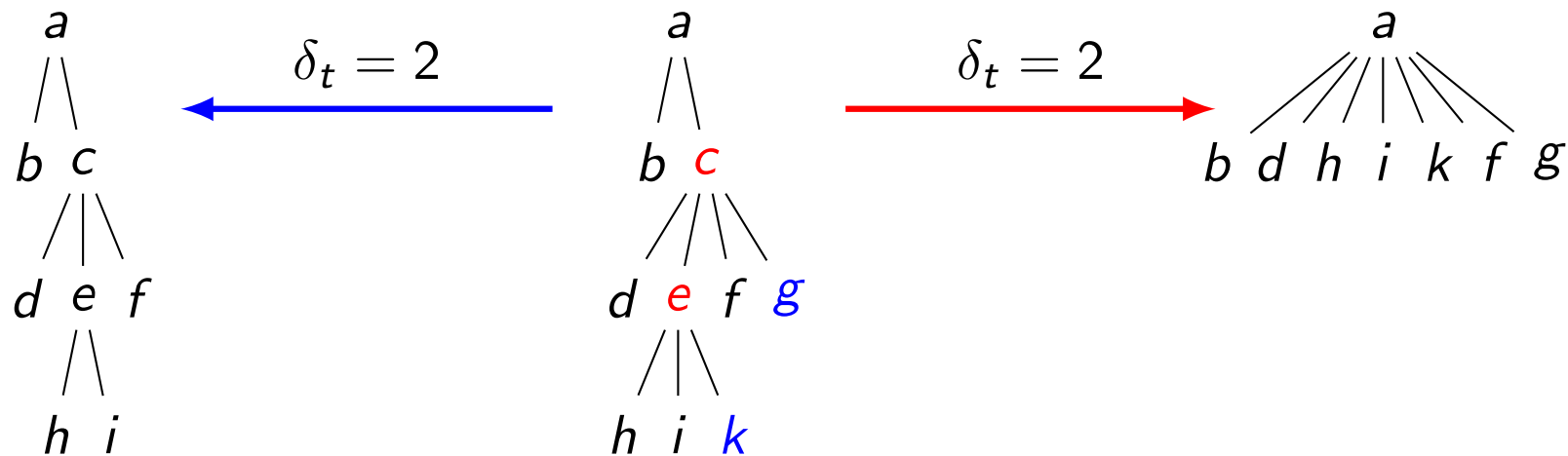
1. We **count** all  $pq$ -grams whose leftmost leaf is a **dummy node**: Each leaf is the anchor node of exactly one  $pq$ -gram whose leftmost leaf is a dummy node, giving  $l$   $pq$ -grams. Each non-leaf is the anchor of  $q - 1$   $pq$ -grams whose leftmost leaf is a dummy, giving  $i(q - 1)$   $pq$ -grams.
2. We **count** all  $pq$ -grams whose leftmost leaf is **not a dummy node**: Each node of the tree except the root is the leftmost leaf of exactly one  $pq$ -gram, giving  $l + i - 1$   $pq$ -grams.

Overall number of  $pq$ -grams:  $l + i(q - 1) + (l + i - 1) = 2l + qi - 1$ .  $\square$

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# Motivation: Unit Cost Model Not Always Intuitive



- Unit cost edit distance:
  - no difference between leaves and non-leaves
  - may lead to non-intuitive results
- **Conclusion:** Non-leaves should have more weight than leaves.

# Fanout Weighted Tree Edit Distance

## Definition (Fanout Weighted Tree Edit Distance)

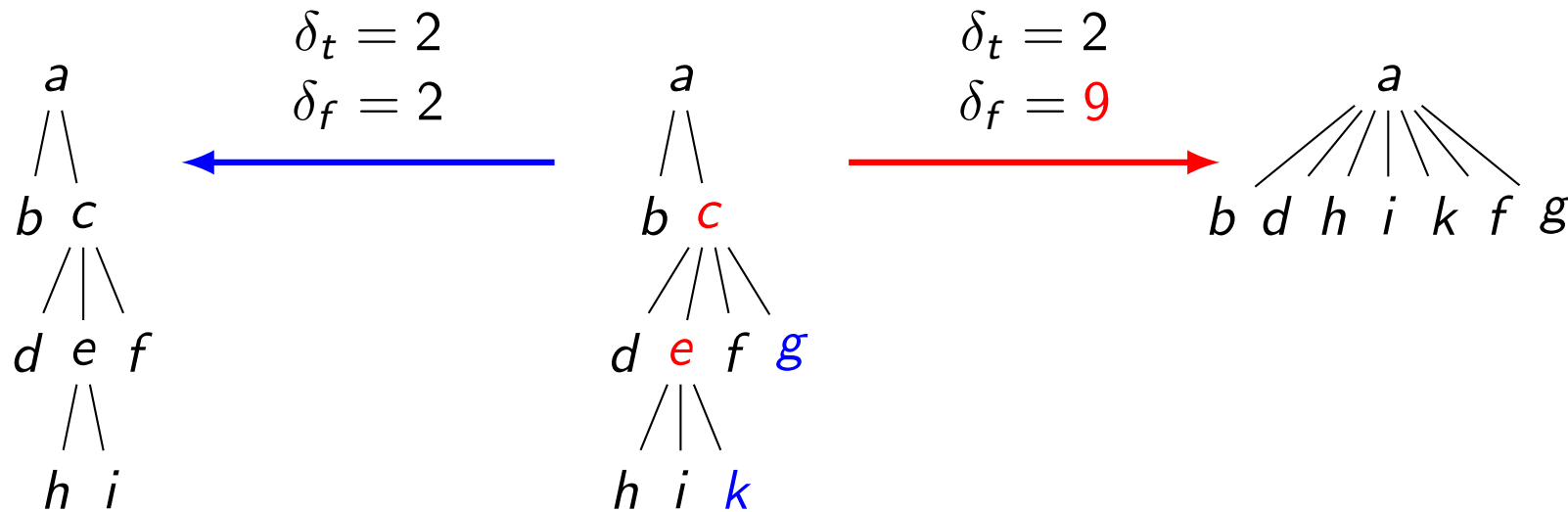
Let  $T$  and  $T'$  be two trees,  $w \in N(T)$  a node with fanout  $f$ ,  $w' \in N(T')$  a node with fanout  $f'$ ,  $c > 0$  a constant. The **fanout weighted tree edit distance**,  $\delta_f = (T, T')$ , between  $T$  and  $T'$  is defined as the tree edit distance with the following costs for the edit operations:

- Delete:  $\alpha(w \rightarrow \epsilon) = f + c$
  - Insert:  $\alpha(\epsilon \rightarrow w') = f' + c$
  - Rename:  $\alpha(w \rightarrow w') = (f + f')/2 + c$
- 
- **Cost** of changing a **non-leaf** node: proportional to its fanout.
  - **Cost** of changing a **leaf** node: constant  $c$ .

# Example: Fanout-Weighted Tree Edit Distance

- Fanout-Weighted Tree Edit Distance:

- leaf changes have small cost ( $c = 1$  in the example)
- non-leaf changes cost proportional to the node fanout





## $pq$ -Gram Distance Lower Bound

### Theorem

Let  $p = 1$  and  $c \geq \max(2q - 1, 2)$  be the cost of changing a leaf node. The  $pq$ -gram distance provides a lower bound for the fanout weighted tree edit distance, i.e., for any two trees,  $T$  and  $T'$ ,

$$\frac{\delta_g(T, T')}{2} \leq \delta_f(T, T').$$

### Proof.

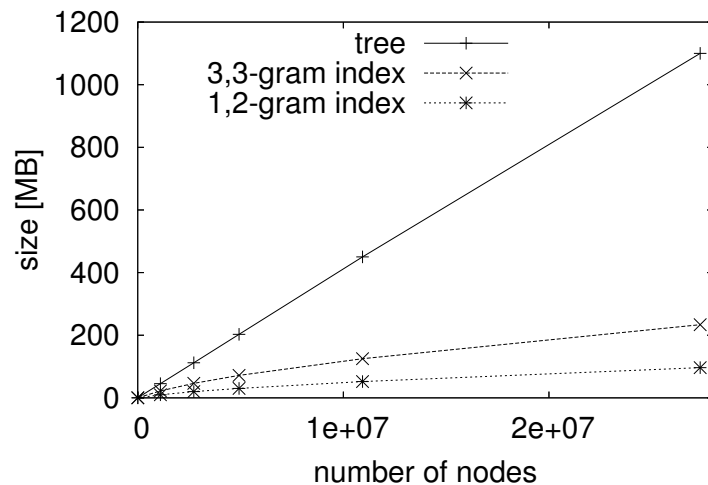
See [ABG10] (ACM Transactions on Database Systems). □

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# Size of the $pq$ -Gram Index

- $pq$ -Gram index size: linear in the tree size
- Experiment:
  - compute  $pq$ -gram index for trees with different number of nodes
  - compare tree and index size



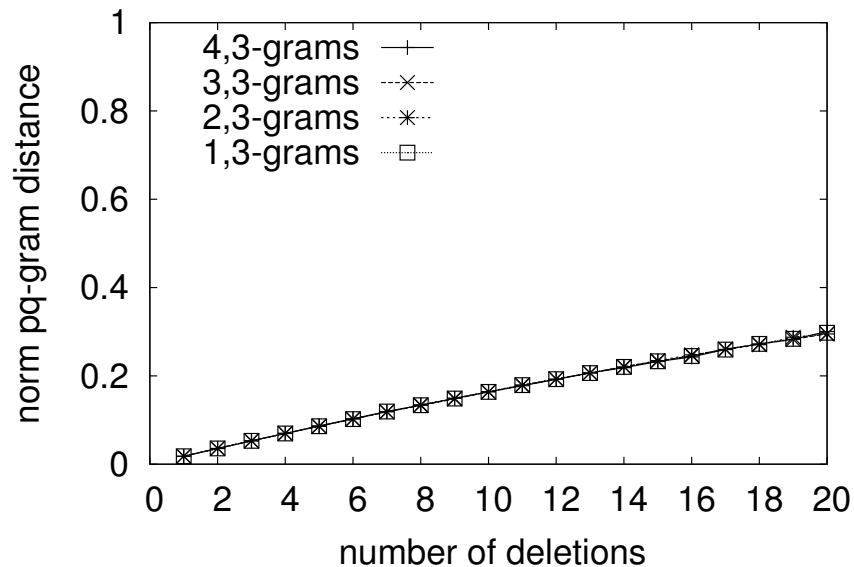
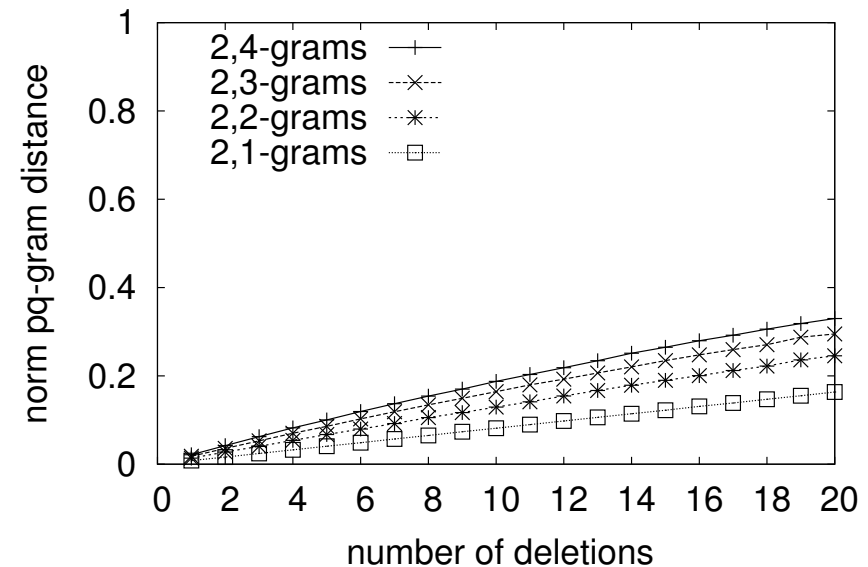
[Trees created with `xmlgen`.]

Why is the  $pq$ -gram index smaller than the tree?

- hash values are smaller than labels
- duplicate  $pq$ -grams of a tree are stored only once

## Sensitivity to Structure Change — Leaf

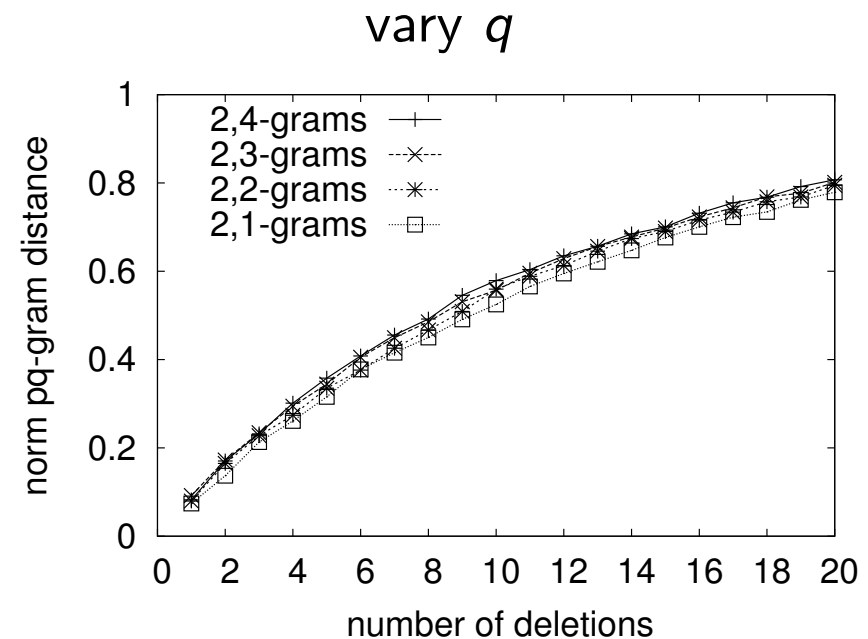
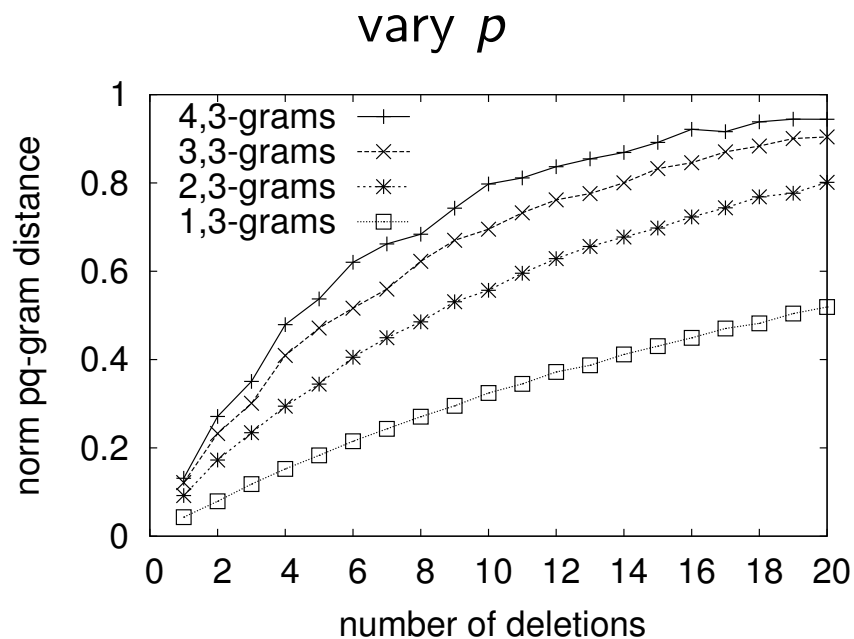
- Cost of leaf change  $\rightarrow$  depends only on  $q$
- Experiment:
  - delete leaf nodes
  - measure normalized  $pq$ -gram distance

vary  $p$ vary  $q$ 

(Artificial tree with 144 nodes, 102 leaves, fanout 2–6 and depth 6. Average over 100 runs.)

## Sensitivity to Structure Change — Non-Leaf

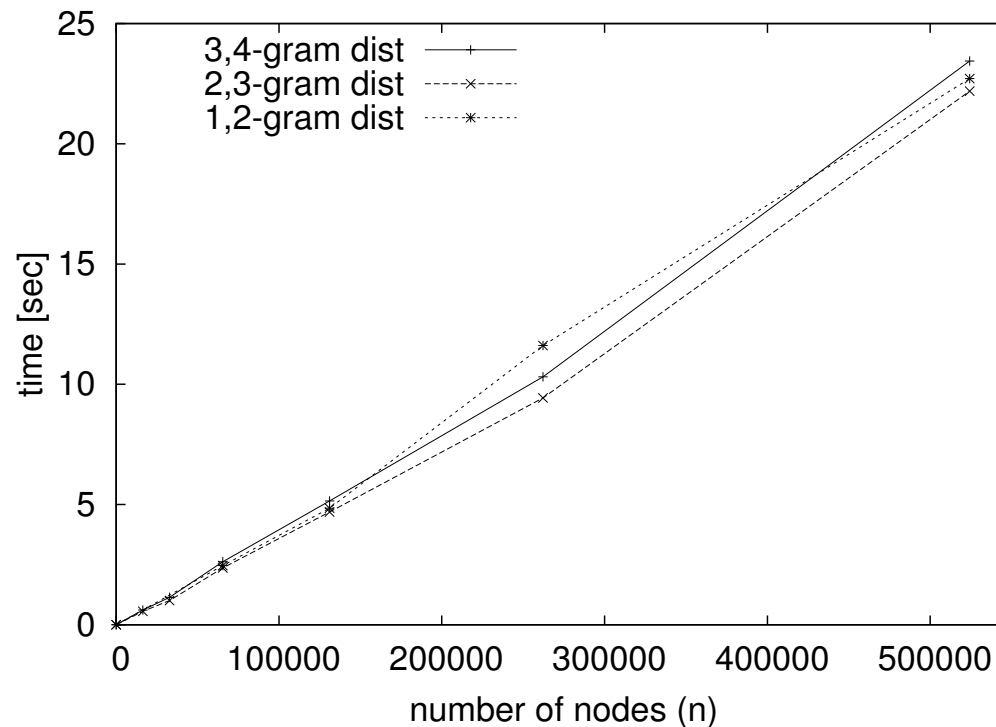
- Cost for non-leaf change  $\rightarrow$  controlled by  $p$
- Experiment:
  - delete non-leaf nodes
  - measure normalized  $pq$ -gram distance



(Artificial tree with 144 nodes, 102 leaves, fanout 2–6 and depth 6. Average over 100 runs.)

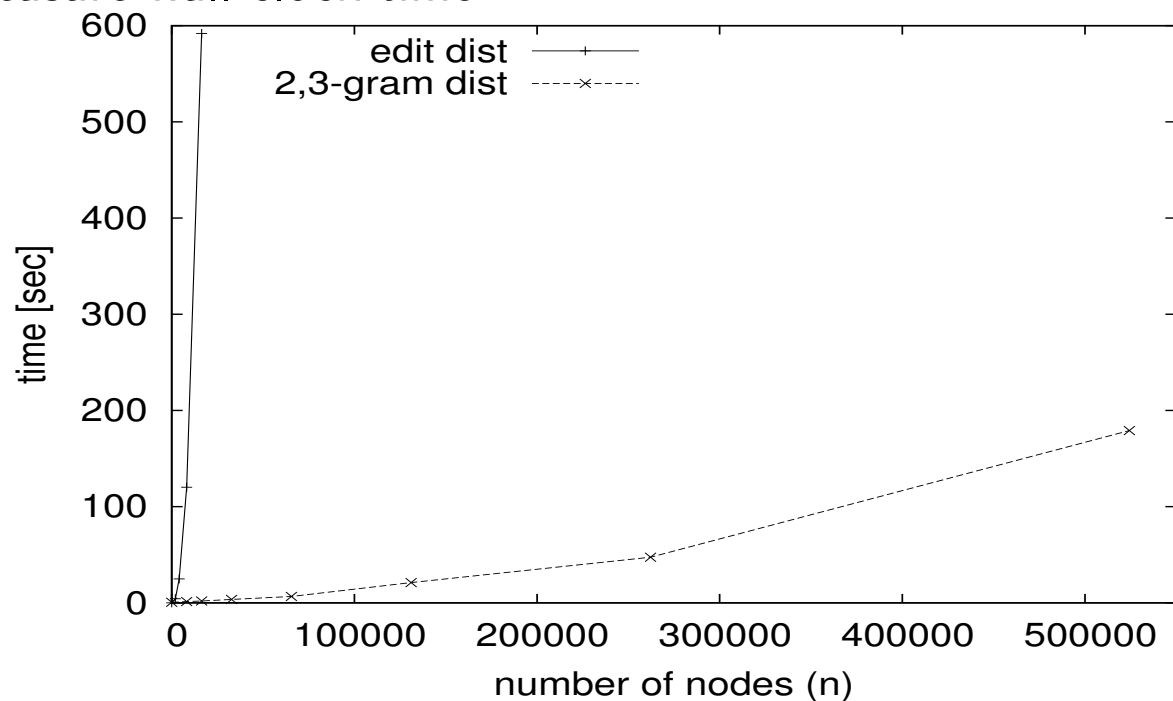
# Influence of $p$ and $q$ on Scalability

- Scalability (almost) independent of  $p$  and  $q$ .
- **Experiment:** For pair of trees
  - compute  $pq$ -gram distance for varying  $p$  and  $q$
  - vary tree size: up to  $10^6$  nodes
  - measure wall clock time



# Scalability to Large Trees

- $pq$ -gram distance  $\rightarrow$  **scalable** to large trees
- compare with edit distance
- **Experiment:** For pair of trees
  - compute tree edit distance and  $pq$ -gram distance
  - vary tree size: up to  $5 \times 10^5$  nodes
  - measure wall clock time



# *$pq$ -Grams vs. other Edit Distance Approximations*

Effectiveness:  $pq$ -grams outperform all other approximations

Experiment: two sets of [address trees](#) (299 and 302 trees)

- compute distances between all tree pairs
- find matches (symmetric nearest neighbor)

Distance	Correct	Recall	Precision	f-Measure	Runtime
fanout edit dist	259	86.6%	98.5%	0.922	19 min
unit edit dist	247	82.6%	96.5%	0.890	14 min
node intersection	197	65.9%	93.8%	0.774	4.3s
$p,q$ -grams	236	78.9%	98.7%	0.877	8.1s
tree-embedding	206	68.9%	96.3%	0.803	7.1s
binary branch	193	64.5%	93.2%	0.763	7.4s
bottom-up	148	49.6%	92.5%	0.645	67.0s



# Summary

- Binary Branch Distance
  - lower bound of the **unit cost** tree edit distance
  - trees are split into binary branches (small subgraphs)
  - similar trees have many common binary branches
  - complexity  $O(n \log n)$  time
- *pq*-Gram Distance
  - lower bound for the **fanout weighted** tree edit distance
  - trees are split into *pq*-grams (small subtrees)
  - similar trees have many common *pq*-grams
  - complexity  $O(n \log n)$  time



Nikolaus Augsten, Michael Böhlen, and Johann Gamper.  
Approximate matching of hierarchical data using  $pq$ -grams.  
*In Proceedings of the International Conference on Very Large Databases (VLDB)*, pages 301–312, Trondheim, Norway, September 2005. ACM Press.



Nikolaus Augsten, Michael Böhlen, and Johann Gamper.  
The  $pq$ -gram distance between ordered labeled trees.  
*ACM Transactions on Database Systems (TODS)*, 35(1):1–36, 2010.



Richard M. Karp and Michael O. Rabin.  
Efficient randomized pattern-matching algorithms.  
*IBM Journal of Research and Development*, 31(2):249–260, March 1987.