Similarity Search
Set Similarity Join

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WS 2023/24

Version January 10, 2024

Outline

Outline

Application Scenario

- Scenario: A social network company stores user interests.
- Example: user table with interests:

R

id	name	interests
s	Sebastian	{bouldering, biking, swimming}
n	Nathan	{bouldering, swimming, guitar, singing}
p	Philippides	{hiking, running}
m	Maria	{bouldering, hiking, running}
r	Rosa	{bouldering, skiing, hiking}
...	...	...

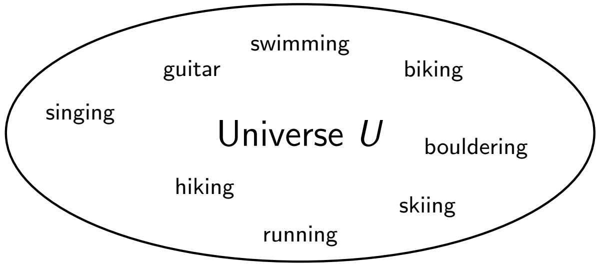
- Task: Recommend new friends based on similar interests!

Notation

collection

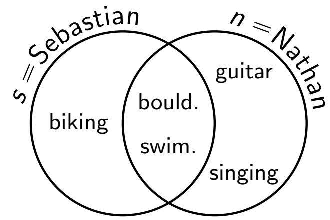
token

$$R = \{\underbrace{\{\text{bouldering}, \text{biking}, \text{swimming}\}}_{\text{set}}, \dots\}$$



Measuring Similarity of Sets

- Goal: measure the similarity of two sets r, s
- Similarity Function:
  - $\text{Sim}(r, s)$ is *high* for similar sets, *low* for dissimilar sets
  - *Example:* Overlap $|r \cap s|$
- Distance Function:
  - $\text{Dis}(r, s)$ is *low* for similar sets, *high* for dissimilar sets
  - *Example:* Hamming distance $|r \triangle s| = |(r \setminus s) \cup (s \setminus r)| = |r \cup s| - |r \cap s|$
- Example:



$|s \cap n| = 2$

$|s \triangle n| = 3$

The Join Approach

- Solution: Compute the set similarity join

Definition (Set Similarity Join)

Given two collections of sets R and S, the set similarity join computes

$$R \tilde{\bowtie} S = \{(r, s) \in R \times S \mid \text{Sim}(r, s) \geqslant t\}$$

for a similarity function Sim or

$$R \tilde{\bowtie} S = \{(r, s) \in R \times S \mid \text{Dis}(r, s) \leqslant t\}$$

for a distance function Dis and threshold t.

- Naive Approach:
  1. Compute all pairs $R \times S$
  2. Test if $\text{Sim}(r, s) \geqslant t$ or $\text{Dis}(r, s) \leqslant t$ on each tuple

Naive Join Example

- Example: self-join $R \tilde{\bowtie} R$, overlap similarity, threshold $t = 2$

R

id	name	interests
s	Sebastian	{bouldering, biking, swimming}
n	Nathan	{bouldering, swimming, guitar, singing}
p	Philippides	{hiking, running}
m	Maria	{bouldering, hiking, running}
r	Rosa	{bouldering, skiing, hiking}

$$R \tilde{\bowtie} R = \{(s, n), (p, m), (m, r)\}$$

- 10 (non-reflexive, non-symmetric) comparisons!

Demonstration

- Experiment: Naive approach
  - self-join with varying $|R|$
  - average set size 10
  - universe size 1000, uniformly distributed
  - overlap similarity with threshold $t = 4$

$\lvert R\rvert$	#comparisons	runtime [s]
1000	$5 \cdot 10^5$	0.022
10000	$5 \cdot 10^7$	2.288
100000	$5 \cdot 10^9$	218.773

- A *single* similarity computation is fast (≈150 CPU cycles)
- But the search space grows fast: $\Theta(|R|^2)$

Outline

Reducing the Search Space using Filters

- Filtering: Reduce the search space by removing dissimilar pairs of sets
- Set similarity Join: Most filters are *signature*-based

Definition (Signature Scheme)

A signature scheme Sign is a function that maps a set of tokens to a set of signatures such that for any two sets of tokens, r and s:

$$\mathrm{Sim}(r, s) \geqslant t \Rightarrow \mathrm{Sign}(r) \cap \mathrm{Sign}(s) \neq \varnothing$$

for a similarity function Sim and

$$\mathrm{Dis}(r, s) \leqslant t \Rightarrow \mathrm{Sign}(r) \cap \mathrm{Sign}(s) \neq \varnothing$$

for a distance function Dis.

- Intuition: Similar sets share at least one signature.

Signature-based Set Similarity Join

- Idea: Similar sets share signatures.
  1. Find all pairs sharing signatures (*candidates*)
  2. Test if $\mathrm{Sim}(r, s) \geqslant t$ or $\mathrm{Dis}(r, s) \leqslant t$ on each tuple
- How do we find pairs sharing signatures?
  1. Compute all pairs $R \times S$
  2. Test if $\mathrm{Sign}(r) \cap \mathrm{Sign}(s) \neq \varnothing$ on each tuple
- Likely slower than naive approach!
- Index: Build a simple index to find sets for each signature

Inverted-list Index

- Inverted-list Index: Stores mappings from content (e.g., signatures) to locations (e.g., sets)
  1. Compute signatures $\text{Sign}(s)$ for set s
  2. Store a pointer to s in the list I_{sig} of each signature $sig \in \text{Sign}(s)$
- Example:

$$R = \{s_1, s_2, s_3, s_4\}$$
$$\text{Sign}(s_1) = \{\star, \spadesuit\}$$
$$\text{Sign}(s_2) = \{\circ, \clubsuit, \heartsuit\}$$
$$\text{Sign}(s_3) = \{\circ, \spadesuit, \diamondsuit\}$$
$$\text{Sign}(s_4) = \{\clubsuit, \spadesuit, \heartsuit, \diamondsuit\}$$

inverted list index

$\star$	$\circ$	$\clubsuit$	$\spadesuit$	$\heartsuit$	$\diamondsuit$
↓	↓	↓	↓	↓	↓
s_1	s_2	s_2	s_1	s_2	s_3
	s_3	s_4	s_3	s_4	s_4
			s_4		

- A good signature scheme is both easy to compute and results in few false positives (= number of unnecessary verifications).

Signature-based Framework

Algorithm 1: Signature-based Framework

Data: Collection R, threshold t
Result: All similar pairs $M \subseteq R \times S$
$I \leftarrow \varnothing$ // inverted list index
forall $s \in S$ **do** // indexing
  forall signatures $sig \in \text{Sign}(s)$ **do**
    $I_{sig} \leftarrow I_{sig} \cup \{s\}$
$M \leftarrow \varnothing$, $C \leftarrow \varnothing$
forall $r \in R$ **do** // probing
  forall signatures $sig \in \text{Sign}(r)$ **do**
    $C \leftarrow C \cup \{(r, s) \mid s \in I_{sig}\}$
forall candidate pairs $(r, s) \in C$ **do**
  $M \leftarrow M \cup (r, s)$ if $\text{Sim}(r, s) \geq t$ (or $\text{Dis}(r, s) \leq t$)
return M

Outline

Identity Signature

- Simplest signature scheme (for overlap) is identity (Sign = Id):

$$|r \cap s| \geq t \Rightarrow \text{Id}(r) \cap \text{Id}(s) \neq \varnothing$$
$$\Leftrightarrow |r \cap s| \geq t \Rightarrow |r \cap s| \geq 1 \qquad \text{assuming } t \geq 1$$

- Every token is a signature
- Example:

id	interests
s	{**B**ould., b**I**king, s**W**im.}
n	{**B**ould., s**W**im., **G**uitar, **S**ing.}
p	{**H**iking, **R**unning}
m	{**B**ould., **H**iking, **R**unning}
r	{**B**ould., s**K**iing, **H**iking}

inverted list index

B	I	W	G	S	H	R	K
↓	↓	↓	↓	↓	↓	↓	↓
s	s	s	n	n	p	p	r
n		n			m	m	
m					r		
r							

Identity Signature

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$$|r \cap s| \geqslant t \Rightarrow \mathrm{Id}(r) \cap \mathrm{Id}(s) \neq \emptyset$$
$$\Leftrightarrow |r \cap s| \geqslant t \Rightarrow |r \cap s| \geqslant 1 \qquad \text{assuming } t \geqslant 1$$

- Every token is a signature
- Example:

id	interests
s	{B, I, W}
n	{B, W, G, S}
p	{H, R}
m	{B, H, R}
r	{B, K, H}

inverted list index

B	I	W	G	S	H	R	K
↓	↓	↓	↓	↓	↓	↓	↓
s	s	s	n	n	p	p	r
n		n			m	m	
m					r		
r							

Identity Signature Example

id	interests
s	{B, I, W}
n	{B, W, G, S}
p	{H, R}
m	{B, H, R}
r	{B, K, H}

inverted list index

B	I	W	G	S	H	R	K
↓	↓	↓	↓	↓	↓	↓	↓
s	s	s	n	n	p	p	r
n		n			m	m	
m					r		
r							

- Probing:
  1. *Set s:* $C \leftarrow \emptyset \cup \{(s, n), (s, m), (s, r)\}$
  2. *Set n:* $C \leftarrow C \cup \{(n, m), (n, r)\}$
  3. *Set p:* $C \leftarrow C \cup \{(p, m), (p, r)\}$
  4. *Set m:* $C \leftarrow C \cup \{(m, r)\}$
- 8 (non-reflexive, non-symmetric) comparisons!
- Most candidates are the result of the long list B.

Demonstration

- Experiment: Identity signature[1]
  - self-join with varying $|R|$
  - average set size 10
  - universe sizes $|U| = 1000$ and $|U| = 10000$, uniformly distributed
  - overlap similarity with threshold $t = 4$

$\|R\|$	$\|U\|$	#comparisons	runtime [s]
1000	1000	$4.7 \cdot 10^4$	0.0
	10000	$4.8 \cdot 10^3$	0.0
10000	1000	$4.7 \cdot 10^6$	0.01
	10000	$5 \cdot 10^5$	0.005
100000	1000	$4.7 \cdot 10^8$	1.6
	10000	$4.9 \cdot 10^7$	0.19
1000000	1000	$4.7 \cdot 10^{10}$	241
	10000	$4.9 \cdot 10^9$	23

- Large improvement over naive approach
- Universe size heavily influences performance

[1] Implementation includes some optimizations compared to framework

Prefix Signature

- Idea: Exploit token order to construct a signature that is based on a *subset* of tokens.

Definition (Prefix Signature Scheme)

The prefix signature $\mathrm{Pre}(r)$ of a set r for overlap threshold t is constructed as follows:

1. Order the tokens of r by any fixed global[a] order.
2. Each of the first $|r| - t + 1$ tokens in the ordered set is a prefix signature.

[a] global: the same order must be used for all sets

- Example:

Set n, $t = 2$

B	W	G	S

1. order (e.g., alphabetically)

B	G	S	W

2. take first $4 - 2 + 1 = 3$ tokens

B	G	S	W

- $\mathrm{Pre}(n) = \{B, G, S\}$

Prefix Signature Correctness I

Lemma

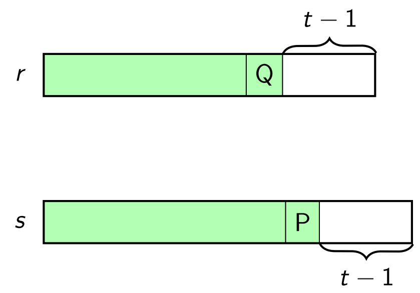
Pre *is a signature scheme for overlap similarity, i.e.,*

$$|r \cap s| \geqslant t \Rightarrow |\operatorname{Pre}(r) \cap \operatorname{Pre}(s)| \geqslant 1$$

We show the contraposition, i.e.,

$$|\operatorname{Pre}(r) \cap \operatorname{Pre}(s)| = 0 \Rightarrow |r \cap s| < t$$

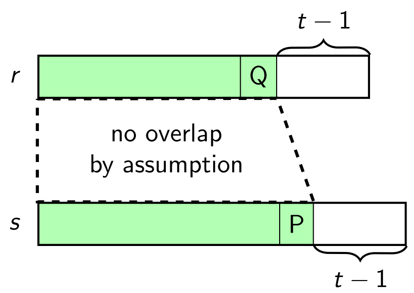
Prefix Signature Correctness II



- Consider sets r, s; Q and P are the largest tokens in the respective prefixes, wlog. assume $P < Q$.
- Assume $\operatorname{Pre}(r) \cap \operatorname{Pre}(s) = \varnothing$. We bound $|r \cap s|$:
  - $|\operatorname{Pre}(r) \cap \operatorname{Pre}(s)| = 0$ by assumption
  - $|(r \setminus \operatorname{Pre}(r)) \cap \operatorname{Pre}(s)| = 0$ as $P < Q$
  - $|r \cap (s \setminus \operatorname{Pre}(s))| \leqslant t - 1$ as $|s \setminus \operatorname{Pre}(s)| = t - 1$
  - Hence, $|r \cap s| < t$.

Filters for the Set Similarity Join | Signatures for Overlap Similarity

Prefix Signature Correctness II



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Filters for the Set Similarity Join | Signatures for Overlap Similarity

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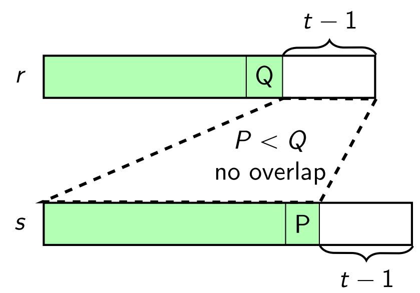

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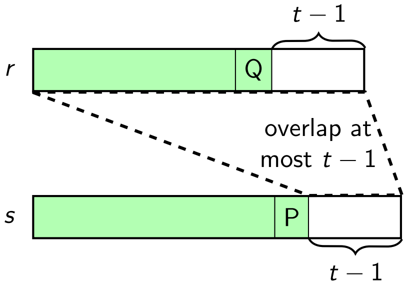
Prefix Signature Correctness II



- Consider sets r, s; Q and P are the largest tokens in the respective prefixes, wlog. assume $P < Q$.
- Assume $\text{Pre}(r) \cap \text{Pre}(s) = \emptyset$. We bound $|r \cap s|$:
  - $|\text{Pre}(r) \cap \text{Pre}(s)| = 0$ by assumption
  - $|(r \setminus \text{Pre}(r)) \cap \text{Pre}(s)| = 0$ as $P < Q$
  - $|r \cap (s \setminus \text{Pre}(s))| \leqslant t-1$ as $|s \setminus \text{Pre}(s)| = t-1$
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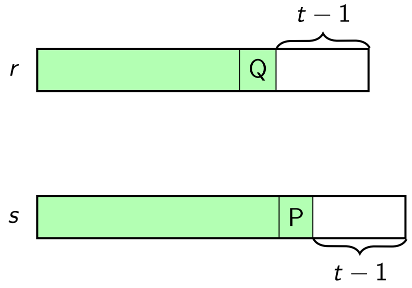
Prefix Signature Correctness II



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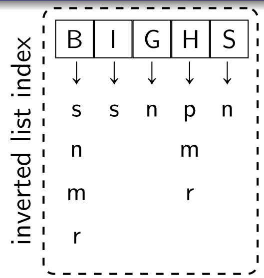
  - $|(r \setminus \text{Pre}(r)) \cap \text{Pre}(s)| = 0$ as $P < Q$
  - $|r \cap (s \setminus \text{Pre}(s))| \leqslant t-1$ as $|s \setminus \text{Pre}(s)| = t-1$
  - Hence, $|r \cap s| < t$.

Prefix Signature Example

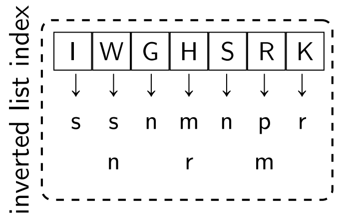
id	interests (ordered alphabetically)
s	{B, I, W}
n	{B, G, S, W}
p	{H, R}
m	{B, H, R}
r	{B, H, K}



- Overlap threshold $t = 2$
- Indexing: all tokens except the last, alphabetical order
- Probing:
  1. *Set s:* $C \leftarrow \emptyset \cup \{(s,n),(s,m),(s,r)\}$
  2. *Set n:* $C \leftarrow C \cup \{(n,m),(n,r)\}$
  3. *Set p:* $C \leftarrow C \cup \{(p,m),(p,r)\}$
  4. *Set m:* $C \leftarrow C \cup \{(m,r)\}$
- still 8 comparisons!
- removing B from the prefix could help

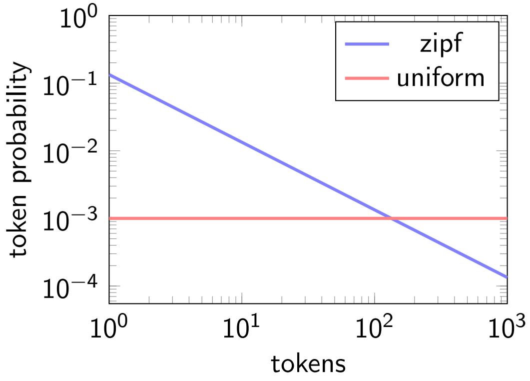
Prefix Signature Example: Order Makes a Difference

id	interests (ordered by frequency)
s	{I, W, B}
n	{G, S, W, B}
p	{R, H}
m	{R, H, B}
r	{K, H, B}



- Overlap threshold $t = 2$
- Indexing: all tokens except the last, ordered by ascending frequency
- Probing:
  1. *Set s:* $C \leftarrow \emptyset \cup \{(s,n)\}$
  2. *Set n:* $C \leftarrow C \cup \emptyset$
  3. *Set p:* $C \leftarrow C \cup \{(p,m)\}$
  4. *Set m:* $C \leftarrow C \cup \{(m,r)\}$
- only 3 comparisons!
- Heuristic: Ordering by ascending token frequency reduces candidates

Two Distributions



- Distribution: Real-world set-data often follow a zipfian distribution
- Skew: Some tokens appear frequently, a large number of tokens is uncommon. This favors the prefix signature.

Demonstration

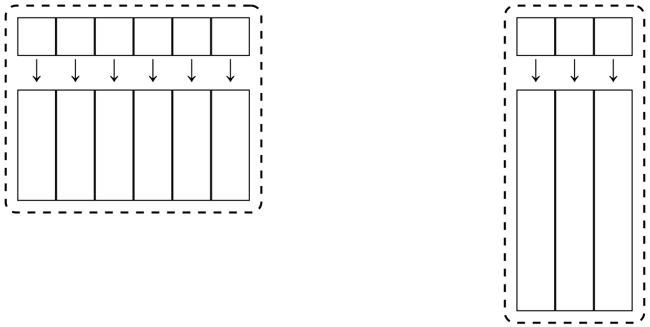
- Experiment: Identity signature vs. Prefix signature[2]
  - self-join with $|R| = 100000$
  - average set size 10
  - universe sizes $|U| = 10000$, uniform and zipfian distribution
  - overlap similarity with threshold $t \in \{4, 6, 8\}$
  - global order: ascending token frequency

Identity				Prefix			
dist.	t	#comp.	runtime [s]	dist.	t	#comp.	runtime [s]
uni.	4	$5.0 \cdot 10^7$	0.187	uni.	4	$2.9 \cdot 10^7$	0.449
	6	$5.0 \cdot 10^7$	0.186		6	$1.8 \cdot 10^7$	0.349
	8	$5.0 \cdot 10^7$	0.186		8	$8.9 \cdot 10^6$	0.145
zipf	4	$3.4 \cdot 10^9$	16.358	zipf	4	$3.1 \cdot 10^8$	3.935
	6	$3.4 \cdot 10^9$	16.862		6	$6.2 \cdot 10^7$	0.873
	8	$3.4 \cdot 10^9$	16.842		8	$1.2 \cdot 10^7$	0.197

[2]Implementation includes some optimizations compared to framework

Impact of Universe Size

- Identity and Prefix: individual tokens used as signatures
- Runtime: proportional to sum of all pairs in each list
- Problem: small universe size reduces filtering effectiveness



- Uniform distribution: halving the universe doubles the list lengths and runtime
- Idea: use a more selective signature than individual tokens

Subset Signature I

Lemma

$$|r \cap s| \geqslant t$$
$$\Leftrightarrow$$
$$\exists p \subseteq U : |p| = t \wedge p \subseteq r \wedge p \subseteq s$$

- Similar sets have at least one common subset of size t. This proves correctness of the following signature:

Definition (Subsets Signature)

The subsets signature $\mathrm{Sub}(r)$ of a set r is defined as:

$$\mathrm{Sub}(r) = \{p \subseteq r \mid |p| = t\}$$

for overlap threshold t.

Subset Signature II

- Sub is stronger than required for signature schemes. It also holds that

$$\mathrm{Sign}(r) \cap \mathrm{Sign}(s) \neq \emptyset \Rightarrow \mathrm{Sim}(r,s) \geqslant t.$$

  Therefore, verification is not necessary.
- For set r and threshold t, we have $|\mathrm{Sub}(r)| = \binom{|r|}{t}$, growing very quickly depending on both $|r|$ and t.
- Example: $n = \{B, W, G, S\}$, $t = 2$

$$\begin{aligned}\mathrm{Sub}(n) = \{&\{B,W\},\{B,G\},\{B,S\}\\ &\{W,G\},\{W,S\},\{G,S\}\}\end{aligned}$$

Demonstration

- Experiment: Prefix signature vs. Subsets signature[3]
  - self-join with $|R| = 1000000$
  - average set size 6
  - universe sizes $|U| \in \{1000, 10000\}$, uniform distribution
  - overlap similarity with threshold $t \in \{3, 4, 5\}$

Prefix			Subsets		
$\lvert U\rvert$	t	runtime [s]	$\lvert U\rvert$	t	runtime [s]
1000	3	336	1000	3	25.6
	4	233		4	41.0
	5	138		5	42.8
10000	3	40.7	10000	3	32.0
	4	24.7		4	53.9
	5	14.9		5	66.2

- Sub outperforms Pre for small set sizes and small universes
- Pre scales better wrt. set size and threshold for large universes

[3] Implementation includes some optimizations compared to framework

Outline

Partitioning I

Definition (Partition)

A partition P of universe U is a family of sets $P = \{p_1, \dots, p_n\}$ with the following properties:

1. $\emptyset \notin P$
2. $\bigcup_{p \in P} p = U$
3. For any $p_i, p_j, i \neq j$, we have $p_i \cap p_j = \emptyset$

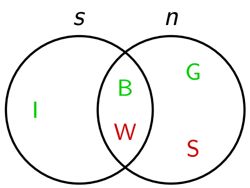
Lemma

For any partition P of universe U and any two sets $r, s \subseteq U$, we have

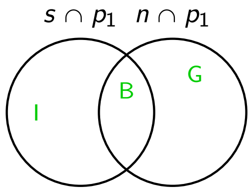
$$Ham(r,s) = \sum_{p \in P} Ham(r \cap p, s \cap p)$$

Partitioning II

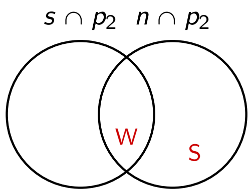
- Example: $P = \{\{B, G, H, I\}, \{K, R, S, W\}\} = \{p_1, p_2\}$



$|s \triangle n| = 3$



$|(s \cap p_1) \triangle (n \cap p_1)| = 2$



$|(s \cap p_2) \triangle (n \cap p_2)| = 1$

Partition Signature

Definition (Partition Signature)

The partition signature $\text{Par}(r)$ of a set r is constructed as follows:

1. Fix any partition P of U into $t + 1$ parts (for Hamming distance t)
2. $\text{Par}(r) = \{(r \cap p_i, i) \mid p_i \in P\}$

- Example: $P = \{\{B, G, H, I\}, \{K, R, S, W\}\} = \{p_1, p_2\}$
  - $\text{Par}(\{I, B, W\}) = \{(\{I, B\}, 1), (\{W\}, 2)\}$
  - $\text{Par}(\{S, W\}) = \{(\emptyset, 1), (\{S, W\}, 2)\}$

Correctness of the Partition Signature

Lemma (Correctness of Par)

$$Ham(r, s) \leqslant t \Rightarrow \text{Par}(r) \cap \text{Par}(s) \neq \emptyset$$

We show the contraposition $\text{Par}(r) \cap \text{Par}(s) = \emptyset \Rightarrow Ham(r, s) > t$

Proof.

Assume $\text{Par}(r) \cap \text{Par}(s) = \emptyset$.

- For any $p_i \in P$, if $r \cap p_i \neq s \cap p_i$, then $Ham(r \cap p_i, s \cap p_i) \geqslant 1$.
- Hence, $Ham(r, s) = \sum_{p \in P} Ham(r \cap p, s \cap p) \geqslant |P| = t + 1 > t$.

□

Demonstration

- Experiment: Prefix signature[4] vs. Partition signature
  - self-join with $|R| = 1000000$
  - average set size 20
  - universe sizes $|U| \in \{1000, 10000\}$, uniform distribution
  - Hamming distance with threshold $t \in \{3, 4, 5\}$

Prefix $\lvert U\rvert$	Prefix t	Prefix runtime [s]	Partition $\lvert U\rvert$	Partition t	Partition runtime [s]
1000	3	326	1000	3	4
	4	451		4	23
	5	576		5	144
10000	3	34	10000	3	4
	4	46		4	23
	5	70		5	144

- Par outperforms Pre for large set sizes and small universes
- Par is less sensitive to universe size compared to Pre
- Pre works better for large universes and small set sizes

[4] Adapted to work with Hamming distance

The Empty Par Signature

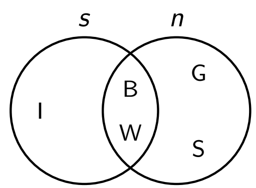
- Consider partitioning $P = \{\{A, B, C\}, \{D, E, F\}, \{G, H, I\}\}$, $t = 2$:

	A	B	C	D	E	F	G	H	I
r	1	1					1	1	
s	1		1				1		1
u		1	1					1	1

- Although all pairs of sets are at Hamming distance 4, they all share the red signature.
- Hence, all pairs of sets are candidates!
- This can happen for small sets or heavily skewed distributions.
- More sophisticated partitioning and more flexible searching in each partition can remedy this problem.

Enumeration: Idea

- How can we make s and n the same?



- By removing I from s and both S and G from n.
- Idea: By removing at most t tokens, all pairs of sets in Hamming distance t can be made equal.

Enumeration Signature

Definition (Enumeration Signature)

The enumeration signature $\text{En}(r)$ of a set r for Hamming distance with threshold t is given by:

$$\text{En}(r) = \{p \subseteq r \mid |p| \geqslant |r| - t\}$$

- Example: $t = 2$
  - $\text{En}(\{I, B, G\}) = \{\{I, B, G\}, \{I, B\}, \{I, G\}, \{B, G\}, \{I\}, \{B\}, \{G\}\}$
  - $\text{En}(\{H, R\}) = \{\{H, R\}, \{H\}, \{R\}, \varnothing\}$

Lemma (Correctness of En)

$$Ham(r, s) \leqslant t \Rightarrow \text{En}(r) \cap \text{En}(s) \neq \varnothing$$

Correctness of the Enumeration Signature

Proof.

- Assume $Ham(r, s) = |(r \backslash s) \cup (s \backslash r)| \leqslant t$.
- Hence, $|r \backslash (r \backslash s)| \geqslant |r| - t$ and $|s \backslash (s \backslash r)| \geqslant |s| - t$
- Consider the set $r \backslash (r \backslash s) = r \cap s = s \backslash (s \backslash r)$
- As $r \cap s \subseteq r$ and $|r \cap s| \geqslant |r| - t$, $r \cap s \in \text{En}(r)$.
- Similarly, $r \cap s \in \text{En}(s)$.
- So $r \cap s \in \text{En}(r) \cap \text{En}(s)$.

$\square$

Demonstration

- Experiment: Partition signature vs. Enumeration signature[5]
  - self-join with $|R| = 1000000$
  - average set size $|r| \in \{8, 16\}$
  - universe size $|U| = 10000$, uniform distribution
  - Hamming distance with threshold $t \in \{1, 2, 3\}$

Partition

$\|r\|$	t	runtime [s]
8	1	4
	2	141
	3	1034
16	1	1
	2	3
	3	14

Enumeration

$\|r\|$	t	runtime [s]
8	1	12
	2	42
	3	165
16	1	21
	2	170
	3	(out of memory)

- En can outperform Par for small thresholds and set sizes
- For large thresholds and sets, En generates too many signatures

[5] Implemented using an optimization called *asymmetric signature scheme* that avoids false positives.

Implementations of Set Similarity Joins

Real implementations of set similarity join algorithms typically

- also support similarity functions other than overlap and Hamming
- use a combination of multiple signature schemes
- extend the algorithmic framework to optimize for their signature schemes
- use additional filters (e.g., based on set *length* or the *position*s of matching signatures)

Outline

Other Similarity Functions

- Normalization: often, normalized similarity functions are preferred
  - $r = \{A, B, C\}, s = \{A, B\}, u = \{A, B, C, D, E, F, G, H, I\}$
  - The pair (r, u) has higher overlap than the pair (r, s)
  - Still, (r, s) might appear more similar due to fewer different tokens
  - Normalizations also consider set sizes and take values in $[0, 1]$
- Jaccard: $\text{Jac}(r, s) = \frac{|r \cap s|}{|r \cup s|} = \frac{|r \cap s|}{|r|+|s|-|r \cap s|}$
- Dice: $\text{Dice}(r, s) = \frac{2|r \cap s|}{|r|+|s|}$
- Cosine: $\text{Cos}(r, s) = \frac{|r \cap s|}{\sqrt{|r| \cdot |s|}}$
- Example:

$$\text{Jac}(r, s) = \frac{|\{A, B\}|}{|\{A, B, C\}|} = \frac{2}{3}$$

$$\text{Jac}(r, u) = \frac{|\{A, B, C\}|}{|\{A, B, C, D, E, F, G, H, I\}|} = \frac{3}{9} = \frac{1}{3}$$

Adapting the Prefix Signature for Jaccard

- The prefix signature operates with overlap similarity
- Idea: Bound minimum overlap s.t. two sets r, s can have $\text{Jac}(r,s) \geqslant t$

$$\begin{aligned} & \frac{|r \cap s|}{|r| + |s| - |r \cap s|} \geqslant t \\ \Leftrightarrow \quad & |r \cap s| \geqslant t(|r| + |s| - |r \cap s|) \\ \Leftrightarrow \quad & |r \cap s| \geqslant \frac{t}{1+t}(|r| + |s|) =: \text{eqo}_J(r,s) \end{aligned}$$

- eqo_J depends on the sizes of two sets. As we want to handle all possible size combinations, we have to get rid of one of them.
- Possible Solution: Assume minimal value $|s| = 1$, but better bounds are possible.

Length Bounds for Jaccard

Lemma

If $\text{Jac}(r,s) \geqslant t$, then $t|r| \leqslant |s| \leqslant \frac{|r|}{t}$.

Lemma

If $\text{Jac}(r,s) \geqslant t$, then

$$\begin{aligned} |r \cap s| &\geqslant \text{eqo}_J(r,s) \\ &\geqslant \text{eqo}_J(r, t|r|) \\ &= t|r| \end{aligned}$$

- Hence, for each set r use $\lceil t|r| \rceil$ as the overlap and proceed as in Pre.

Outline

Algorithm	Signature[6]	Remarks
AllPairs [BMS07]	Length+ Pre	
PPJoin [XWLY08]	Length+ Pre	additional filter based on position of prefix matches
SkipJoin [WQL+19]	Length+ Pre	tighter length filter using prefix positions, removes unmatchable entries from index, can leverage knowledge of set similarity "transitively"
SizeAware [DTL18]	Sub (small sets)+ Id (large sets)	avoids enumerating all subsets in Sub by exploiting an order on subsets, handles small and large sets differently
PartEnum [AGK06]	Length+ Par + En	partitions sets into smaller subsets, enumerates in each partition
PartAlloc [DLWF15]	Length+ Par + En	more flexible than PartEnum, has tighter filtering condition
GPH [QXW+21]	Par + En	more flexible than PartAlloc, optimizes how partitions are chosen

[6] Refers to the closest signature discussed during the lecture; the listed algorithms often use a more efficient variation of the respective signatures.

Summary

- Naive set similarity join inefficient due to large search space
- Signature-based filters speed up join:
  - Id and Pre: single tokens as signatures
  - Sub: all subsets of overlap size as signatures
  - Par: non-overlapping subsets as signatures
  - En: all subsets in Hamming distance as signatures
- Performance depends on dataset's characteristics

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