Similarity Search

Traversal Strings and Constrained Edit Distance

Nikolaus Augsten
nikolaus.augsten@plus.ac.at
Department of Computer Science
University of Salzburg

database research group
https://dbresearch.uni-salzburg.at

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Outline

1. Search Space Reduction for the Tree Edit Distance
   - Similarity Join and Search Space Reduction
   - Lower Bound: Traversal Strings
   - Upper Bound: Constrained Edit Distance
2. Conclusion

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Definition: Similarity Join

Definition (Similarity Join)

Given two sets of trees, S_1 and S_2, and a distance threshold τ, let $\delta_t(\mathrm{T}_i, \mathrm{T}_j)$ be a function that assesses the edit distance between two trees $\mathrm{T}_i \in S_1$ and $\mathrm{T}_j \in S_2$. The similarity join operation between two sets of trees reports in the output all pairs of trees $(\mathrm{T}_i, \mathrm{T}_j) \in S_1 \times S_2$ such that $\delta_t(\mathrm{T}_i, \mathrm{T}_j) \leq \tau$.

Similarity Join Algorithm with Upper and Lower Bounds

simJoin(S_1, S_2)

```
for each $T_i \in S_1$ do
    for each $T_j \in S_2$ do
        if upperBound($T_i, T_j$) $\leq \tau$ then
            output($T_i, T_j$)
        else if lowerBound($T_i, T_j$) $> \tau$ then
            /* do nothing */
        else if $\delta_t(T_i, T_j) \leq \tau$ then
            output($T_i, T_j$)
```

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Preorder and Postorder Traversal Strings

- Each node label is a single character of an alphabet Σ.
- Traversal Strings:
  - *pre*(T) is the string of T's node labels in preorder
  - *post*(T) is the string of T's node labels in postorder

Lemma (Tree Inequality)

Let $pre(T_1)$ and $pre(T_2)$ be the preorder strings, and $post(T_1)$ and $post(T_2)$ be the postorder strings of two trees T_1 and T_2, respectively. Then

$$pre(T_1) \neq pre(T_2) \text{ or } post(T_1) \neq post(T_2) \Rightarrow T_1 \neq T_2$$

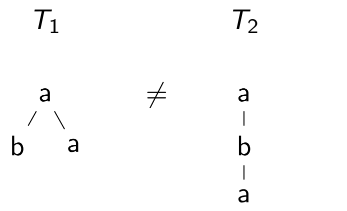
Proof.

The inversion of the argument is obviously true:

$$T_1 = T_2 \Rightarrow pre(T_1) = pre(T_2) \text{ and } post(T_1) = post(T_2) \qquad \square$$

Notes: Traversal Strings and Tree Inequality

- If the traversal strings of two trees are equal, the trees can still be different:



$$pre(T_1) = \text{aba} \quad pre(T_2) = \text{aba}$$

Lower Bound

Theorem (Lower Bound)

If the trees are at tree edit distance k, then the string edit distance between their preorder and postorder traversals is at most k.

Proof.

Tree operations map to string operations (illustration on next slide):

- Insertion ($ins(\mathrm{v}, \mathrm{p}, k, m)$): Let $t_1 \dots t_f$ be the subtrees rooted in the children of p. Then the preorder traversal of the subtree rooted in p is

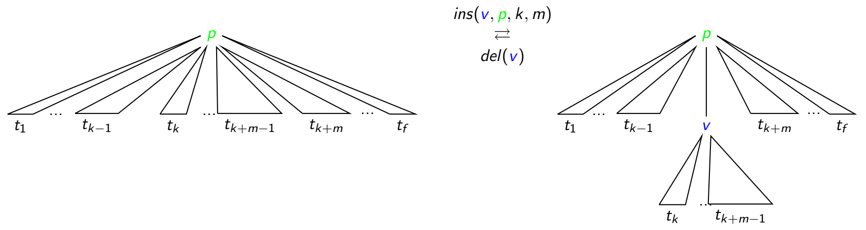
  $$\mathrm{p}\; pre(t_1) \dots pre(t_{k-1}) pre(t_k) \dots pre(t_{k+m-1}) pre(t_{k+m}) \dots pre(t_f).$$

  Inserting v moves the subtrees k to m:

  $$\mathrm{p}\, pre(t_1) \dots pre(t_{k-1})\, \mathrm{v}\; pre(t_k) \dots pre(t_{k+m-1}) pre(t_{k+m}) \dots pre(t_f).$$

  The string distance is 1. Analog rationale for postorder.
- Deletion: Inverse of insertion.
- Rename: With node rename a single string character is renamed. □

Illustration for the Lower Bound Proof (Preorder)



$\mathrm{p}\; pre(\mathrm{t}_1) \dots pre(\mathrm{t}_{k-1})$
$pre(\mathrm{t}_k) \dots pre(\mathrm{t}_{k+m-1})$
$pre(\mathrm{t}_{k+m}) \dots pre(\mathrm{t}_f)$

$\mathrm{p}\; pre(\mathrm{t}_1) \dots pre(\mathrm{t}_{k-1})$
$\mathrm{v}\; pre(\mathrm{t}_k) \dots pre(\mathrm{t}_{k+m-1})$
$pre(\mathrm{t}_{k+m}) \dots pre(\mathrm{t}_f)$

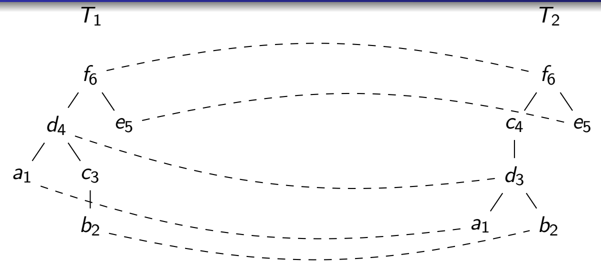
Lower Bound

- From the lower bound theorem it follows that

  $$\max(\delta_s(pre(\mathrm{T}_1), pre(\mathrm{T}_2)), \delta_s(post(\mathrm{T}_1), post(\mathrm{T}_2))) \leq \delta_t(\mathrm{T}_1, \mathrm{T}_2)$$

  where δ_s and δ_t are the string and the tree edit distance, respectively.
- The string edit distance can be computed faster:
  - string edit distance runtime: $O(n^2)$
  - tree edit distance runtime: $O(n^3)$
- Similarity join: match all trees with $\delta_t(\mathrm{T}_1, \mathrm{T}_2) \leq \tau$
  - if $\max(\delta_s(pre(\mathrm{T}_1), pre(\mathrm{T}_2)), \delta_s(post(\mathrm{T}_1), post(\mathrm{T}_2))) > \tau$ then $\delta_t(\mathrm{T}_1, \mathrm{T}_2) > \tau$
  - thus we do not have to compute the expensive tree edit distance

Example: Traversal String Lower Bound



$pre(\mathrm{T}_1) =$ fdacbe

$post(\mathrm{T}_1) =$ abcdef

$pre(\mathrm{T}_2) =$ fcdabe

$post(\mathrm{T}_2) =$ abdcef

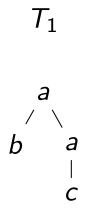
$$\delta_s(pre(\mathrm{T}_1), pre(\mathrm{T}_2)) = 2$$

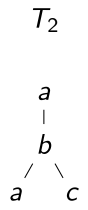
$$\delta_s(post(\mathrm{T}_1), post(\mathrm{T}_2)) = 2$$

$$\delta_t(\mathrm{T}_1, \mathrm{T}_2) = 2$$

Example: Traversal String Lower Bound

- The string distances of preorder and postorder may be different.
- The string distances and the tree distance may be different.





$pre(T_1) = abac$

$post(T_1) = bcaa$

$pre(T_2) = abac$

$post(T_2) = acba$

$\delta_s(pre(T_1), pre(T_2)) = 0$

$\delta_s(post(T_1), post(T_2)) = 2$

$\delta_t(T_1, T_2) = 3$

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Edit Mapping

- Recall the definition of the edit mapping:

Definition (Edit Mapping)

An edit mapping M between T_1 and T_2 is a set of node pairs that satisfy the following conditions:

(1) $(a, b) \in M \Rightarrow a \in N(T_1), b \in N(T_2)$

(2) for any two pairs (a, b) and (x, y) of M:

- (i) $a = x \Leftrightarrow b = y$ (one-to-one condition)
- (ii) a is to the left of x[1] $\Leftrightarrow$ b is to the left of y (order condition)
- (iii) a is an ancestor of x $\Leftrightarrow$ b is an ancestor of y (ancestor condition)

[1] i.e., a precedes x in both preorder and postorder

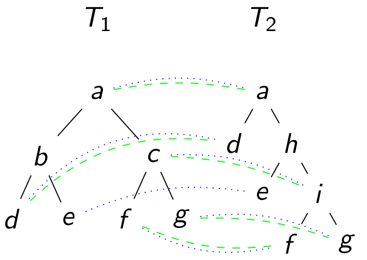
Constrained Edit Distance

- We compute a special case of the edit distance to get a faster algorithm.
- $lca(a, b)$ is the lowest common ancestor of a and b.
- Additional requirement on the mapping M:
  (4) for any pairs (a_1, b_1), (a_2, b_2), (x, y) of M:

$$lca(a_1, a_2) \text{ is a proper ancestor of } x$$
$$\Leftrightarrow$$
$$lca(b_1, b_2) \text{ is a proper ancestor of } y.$$

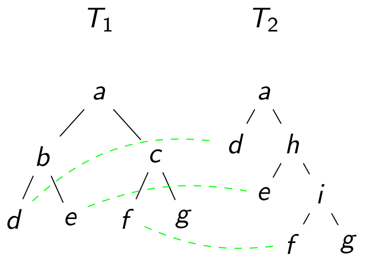
- Intuition: Distinct subtrees of T_1 are mapped to distinct subtrees of T_2.

Example: Constrained Edit Distance



- Constrained edit distance (dashed lines): $\delta_c(\mathrm{T}_1, \mathrm{T}_2) = 5$
  - constrained mapping $M_c = \{(a,a),(d,d),(c,i),(f,f)(g,g)\}$
  - edit sequence: $ren(c,i), del(b), del(e), ins(h), ins(e)$
- Unconstrained edit distance (dotted lines): $\delta_t(\mathrm{T}_1, \mathrm{T}_2) = 3$
  - mapping $M_t = \{(a,a),(d,d),(e,e),(c,i),(f,f)(g,g)\}$
  - edit sequence: $ren(c,i), del(b), ins(h)$

Example: Constrained Edit Distance



- (e,e) violates the $4th$ condition of the constrained mapping:
  - $lca(e,f)$ in T_1 is a
  - a is a proper ancestor of d in T_1
  - assume $(e,e),(f,f),(d,d) \in M_c$
  - $lca(e,f)$ in T_2 is h
  - h is not a proper ancestor of d in T_2

Complexity of the Constrained Edit Distance

Theorem (Complexity of the Constrained Edit Distance)

Let T_1 and T_2 be two trees with $|\mathrm{T}_1|$ and $|\mathrm{T}_2|$ nodes, respectively. There is an algorithm that computes the constrained edit distance between T_1 and T_2 with runtime

$$O(|\mathrm{T}_1||\mathrm{T}_2|).$$

Proof.

See [Zha95, GJK$^+$02]. □

Constrained Edit Distance: Upper Bound

Theorem (Upper Bound)

Let T_1 and T_2 be two trees, let $\delta_t(\mathrm{T}_1, \mathrm{T}_2)$ be the unconstrained and $\delta_c(\mathrm{T}_1, \mathrm{T}_2)$ be the constrained tree edit distance, respectively. Then

$$\delta_t(\mathrm{T}_1, \mathrm{T}_2) \leq \delta_c(\mathrm{T}_1, \mathrm{T}_2)$$

Proof.

See [GJK$^+$02]. □

Use of the Upper Bound

- The constrained edit distance can be computed faster:
  - constrained edit distance runtime: $O(n^2)$
  - unconstrained edit distance runtime: $O(n^3)$
- Similarity join: match all trees with $\delta_t(\mathrm{T}_1, \mathrm{T}_2) \leq \tau$
  - if $\delta_c(\mathrm{T}_1, \mathrm{T}_2) \leq \tau$ then also $\delta_t(\mathrm{T}_1, \mathrm{T}_2) \leq \tau$.
  - thus we do not have to compute the expensive tree edit distance

Summary

- Tree Edit Distance Complexity
- Search Space Reduction
  - Lower Bound: Traversal Strings
  - Upper Bound: Constrained Edit Distance

Sudipto Guha, H. V. Jagadish, Nick Koudas, Divesh Srivastava, and Ting Yu.
Approximate XML joins.
In *Proceedings of the ACM SIGMOD International Conference on Management of Data*, pages 287–298, Madison, Wisconsin, 2002. ACM Press.

Kaizhong Zhang.
Algorithms for the constrained editing distance between ordered labeled trees and related problems.
Pattern Recognition, 28(3):463–474, 1995.