

Similarity Search

Set Similarity Join

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Outline

1 Filters for the Set Similarity Join

- Motivation
- Signature-based Filtering
- Signatures for Overlap Similarity
- Signatures for Hamming Distance

2 Implementations of Set Similarity Joins

- Other Similarity Functions
- Table of Set Similarity Join Algorithms and their Signatures

3 Conclusion

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Application Scenario

- **Scenario:** A social network company stores user interests.
- **Example:** user table with interests:

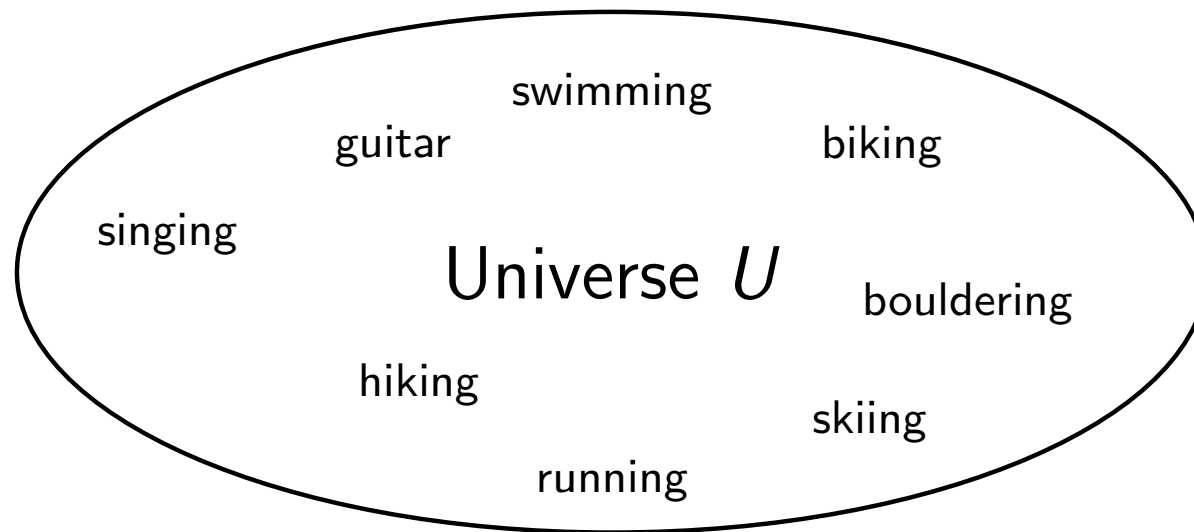
R		
id	name	interests
s	Sebastian	{bouldering, biking, swimming}
n	Nathan	{bouldering, swimming, guitar, singing}
p	Philippides	{hiking, running}
m	Maria	{bouldering, hiking, running}
r	Rosa	{bouldering, skiing, hiking}
...	...	...

- **Task:** Recommend new friends based on similar interests!

Notation

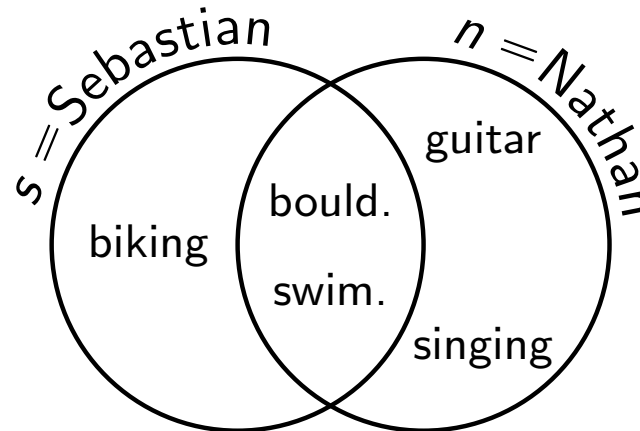
collection

token

$$R = \{\underbrace{\{\text{bouldering, biking, swimming}\}}_{\text{set}}, \dots\}$$


Measuring Similarity of Sets

- **Goal:** measure the similarity of two sets r, s
- **Similarity Function:**
 - $\text{Sim}(r, s)$ is *high* for similar sets, *low* for dissimilar sets
 - *Example:* Overlap $|r \cap s|$
- **Distance Function:**
 - $\text{Dis}(r, s)$ is *low* for similar sets, *high* for dissimilar sets
 - *Example:* Hamming distance $|r \triangle s| = |(r \setminus s) \cup (s \setminus r)| = |r \cup s| - |r \cap s|$
- **Example:**



$$|s \cap n| = 2$$

$$|s \triangle n| = 3$$

The Join Approach

- **Solution:** Compute the set similarity join

Definition (Set Similarity Join)

Given two collections of sets R and S , the set similarity join computes

$$R \bowtie S = \{(r, s) \in R \times S \mid \text{Sim}(r, s) \geq t\}$$

for a similarity function Sim or

$$R \tilde{\bowtie} S = \{(r, s) \in R \times S \mid \text{Dis}(r, s) \leq t\}$$

for a distance function Dis and threshold t .

- **Naive Approach:**

1. Compute all pairs $R \times S$
2. Test if $\text{Sim}(r, s) \geq t$ or $\text{Dis}(r, s) \leq t$ on each tuple

Naive Join Example

- **Example:** self-join $R \tilde{\bowtie} R$, overlap similarity, threshold $t = 2$

R		
id	name	interests
s	Sebastian	{bouldering, biking, swimming}
n	Nathan	{bouldering, swimming, guitar, singing}
p	Philippides	{hiking, running}
m	Maria	{bouldering, hiking, running}
r	Rosa	{bouldering, skiing, hiking}

$$R \tilde{\bowtie} R = \{(s, n), (p, m), (m, r)\}$$

- **10** (non-reflexive, non-symmetric) **comparisons!**

Demonstration

- **Experiment:** Naive approach

- self-join with varying $|R|$
- average set size 10
- universe size 1000, uniformly distributed
- overlap similarity with threshold $t = 4$

$ R $	#comparisons	runtime [s]
1000	$5 \cdot 10^5$	0.022
10000	$5 \cdot 10^7$	2.288
100000	$5 \cdot 10^9$	218.773

- A *single* similarity computation is fast (≈ 150 CPU cycles)
- But the search space grows fast: $\Theta(|R|^2)$

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Reducing the Search Space using Filters

- **Filtering:** Reduce the search space by removing dissimilar pairs of sets
- **Set similarity Join:** Most filters are *signature*-based

Definition (Signature Scheme)

A signature scheme Sign is a function that maps a set of tokens to a set of signatures such that for any two sets of tokens, r and s :

$$\text{Sim}(r, s) \geq t \Rightarrow \text{Sign}(r) \cap \text{Sign}(s) \neq \emptyset$$

for a similarity function Sim and

$$\text{Dis}(r, s) \leq t \Rightarrow \text{Sign}(r) \cap \text{Sign}(s) \neq \emptyset$$

for a distance function Dis .

- **Intuition:** Similar sets share at least one signature.

Signature-based Set Similarity Join

- **Idea:** Similar sets share signatures.
 1. Find all pairs sharing signatures (*candidates*)
 2. Test if $\text{Sim}(r, s) \geq t$ or $\text{Dis}(r, s) \leq t$ on each tuple
- How do we find pairs sharing signatures?
 1. Compute all pairs $R \times S$
 2. Test if $\text{Sign}(r) \cap \text{Sign}(s) \neq \emptyset$ on each tuple
- **Likely slower than naive approach!**
- **Index:** Build a simple index to find sets for each signature

Inverted-list Index

- **Inverted-list Index:** Stores mappings from content (e.g., signatures) to locations (e.g., sets)
 1. Compute signatures $\text{Sign}(s)$ for set s
 2. Store a pointer to s in the list I_{sig} of each signature $sig \in \text{Sign}(s)$
- **Example:**

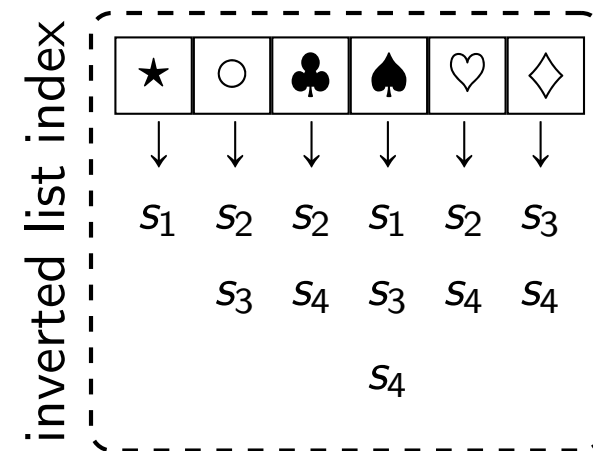
$$R = \{s_1, s_2, s_3, s_4\}$$

$$\text{Sign}(s_1) = \{\star, \spadesuit\}$$

$$\text{Sign}(s_2) = \{\circ, \clubsuit, \heartsuit\}$$

$$\text{Sign}(s_3) = \{\circ, \spadesuit, \diamondsuit\}$$

$$\text{Sign}(s_4) = \{\clubsuit, \spadesuit, \heartsuit, \diamondsuit\}$$



- A good signature scheme is both easy to compute and results in few false positives (= number of unnecessary verifications).

Signature-based Framework

Algorithm 1: Signature-based Framework

Data: Collection R , threshold t

Result: All similar pairs $M \subseteq R \times S$

$I \leftarrow \emptyset$ // inverted list index

forall $s \in S$ **do** // indexing

forall signatures $sig \in \text{Sign}(s)$ **do**

$I_{sig} \leftarrow I_{sig} \cup \{s\}$

$M \leftarrow \emptyset, C \leftarrow \emptyset$

forall $r \in R$ **do** // probing

forall signatures $sig \in \text{Sign}(r)$ **do**

$C \leftarrow C \cup \{(r, s) \mid s \in I_{sig}\}$

forall candidate pairs $(r, s) \in C$ **do**

$M \leftarrow M \cup (r, s)$ if $\text{Sim}(r, s) \geq t$ (or $\text{Dis}(r, s) \leq t$)

return M

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Identity Signature

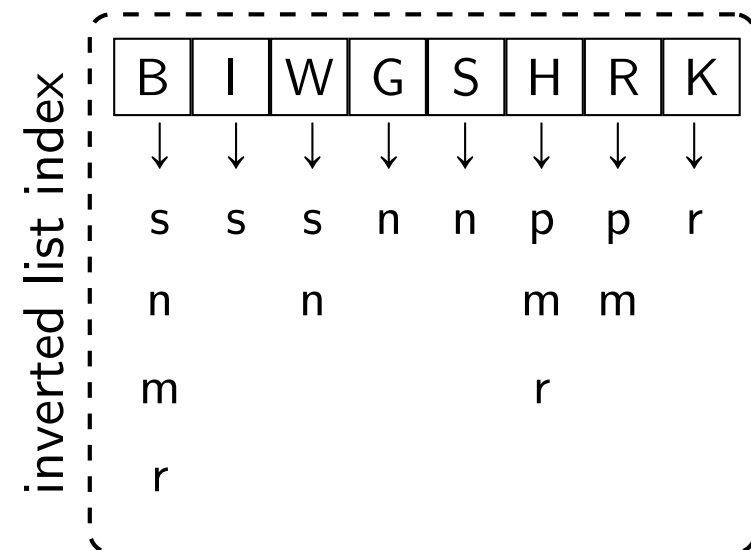
- Simplest signature scheme (for overlap) is identity ($\text{Sign} = \text{Id}$):

$$|r \cap s| \geq t \Rightarrow \text{Id}(r) \cap \text{Id}(s) \neq \emptyset$$

$$\Leftrightarrow |r \cap s| \geq t \Rightarrow |r \cap s| \geq 1 \quad \text{assuming } t \geq 1$$

- Every token is a signature
- Example:

id	interests
s	{ B ould., b I king, s W im.}
n	{ B ould., s W im., G uitar, S ing.}
p	{ H iking, R unning}
m	{ B ould., H iking, R unning}
r	{ B ould., s K iing, H iking}



Identity Signature

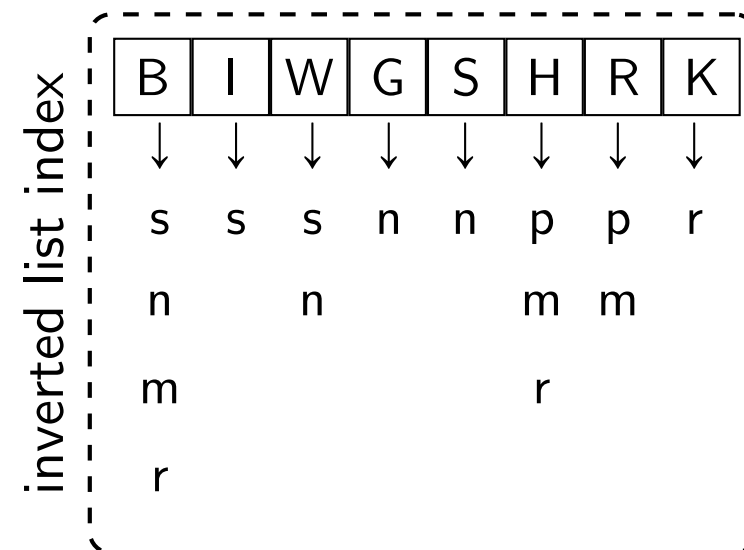
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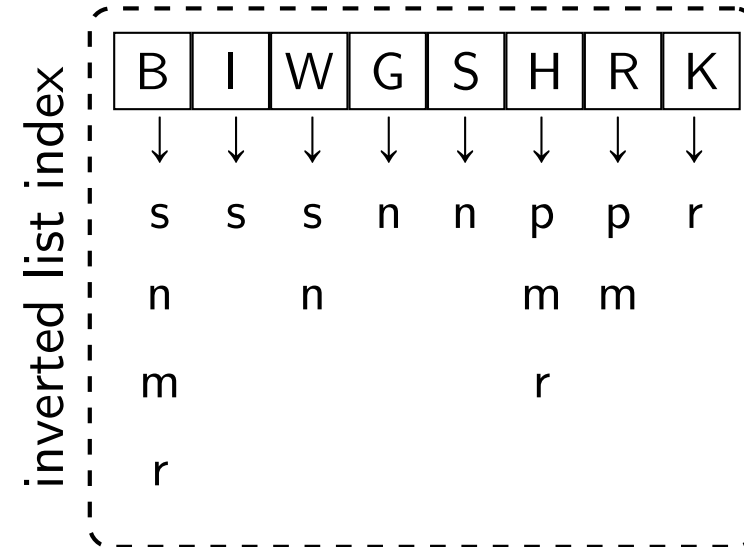
- Every token is a signature
- Example:

id	interests
s	{B, I, W}
n	{B, W, G, S}
p	{H, R}
m	{B, H, R}
r	{B, K, H}



Identity Signature Example

id	interests
s	{B, I, W}
n	{B, W, G, S}
p	{H, R}
m	{B, H, R}
r	{B, K, H}



- Probing:

1. Set s : $C \leftarrow \emptyset \cup \{(s, n), (s, m), (s, r)\}$
2. Set n : $C \leftarrow C \cup \{(n, m), (n, r)\}$
3. Set p : $C \leftarrow C \cup \{(p, m), (p, r)\}$
4. Set m : $C \leftarrow C \cup \{(m, r)\}$

- 8 (non-reflexive, non-symmetric) comparisons!
- Most candidates are the result of the long list B.

Demonstration

- **Experiment:** Identity signature¹
 - self-join with varying $|R|$
 - average set size 10
 - universe sizes $|U| = 1000$ and $|U| = 10000$, uniformly distributed
 - overlap similarity with threshold $t = 4$

$ R $	$ U $	#comparisons	runtime [s]
1000	1000	$4.7 \cdot 10^4$	0.0
	10000	$4.8 \cdot 10^3$	0.0
10000	1000	$4.7 \cdot 10^6$	0.01
	10000	$5 \cdot 10^5$	0.005
100000	1000	$4.7 \cdot 10^8$	1.6
	10000	$4.9 \cdot 10^7$	0.19
1000000	1000	$4.7 \cdot 10^{10}$	241
	10000	$4.9 \cdot 10^9$	23

- Large improvement over naive approach
- Universe size heavily influences performance

¹Implementation includes some optimizations compared to framework

Prefix Signature

- **Idea:** Exploit token order to construct a signature that is based on a *subset* of tokens.

Definition (Prefix Signature Scheme)

The prefix signature $\text{Pre}(r)$ of a set r for overlap threshold t is constructed as follows:

1. Order the tokens of r by any fixed global^a order.
2. Each of the first $|r| - t + 1$ tokens in the ordered set is a prefix signature.

^aglobal: the same order must be used for all sets

- **Example:**

Set n , $t = 2$

B	W	G	S
---	---	---	---

1. order (e.g., alphabetically)

B	G	S	W
---	---	---	---

2. take first $4 - 2 + 1 = 3$ tokens

B	G	S	W
---	---	---	---

- $\text{Pre}(n) = \{B, G, S\}$

Prefix Signature Correctness I

Lemma

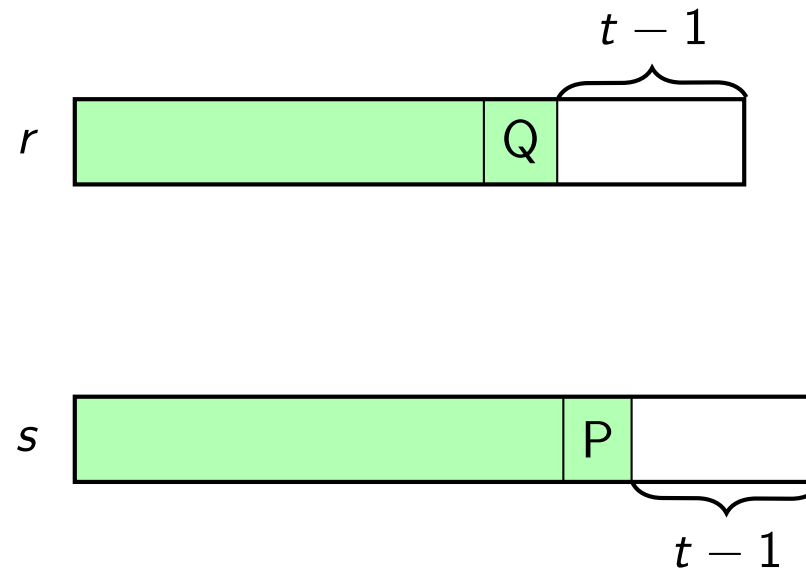
Pre is a signature scheme for overlap similarity, i.e.,

$$|r \cap s| \geq t \Rightarrow |\text{Pre}(r) \cap \text{Pre}(s)| \geq 1$$

We show the contraposition, i.e.,

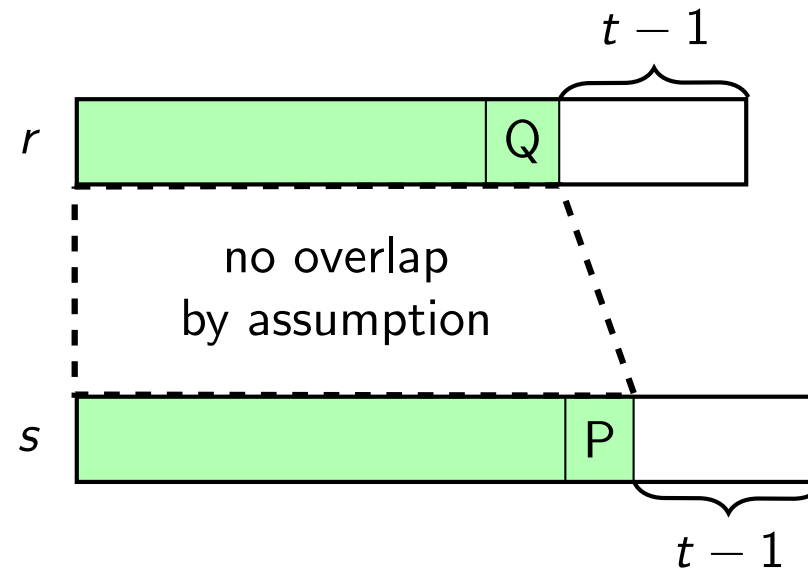
$$|\text{Pre}(r) \cap \text{Pre}(s)| = 0 \Rightarrow |r \cap s| < t$$

Prefix Signature Correctness II



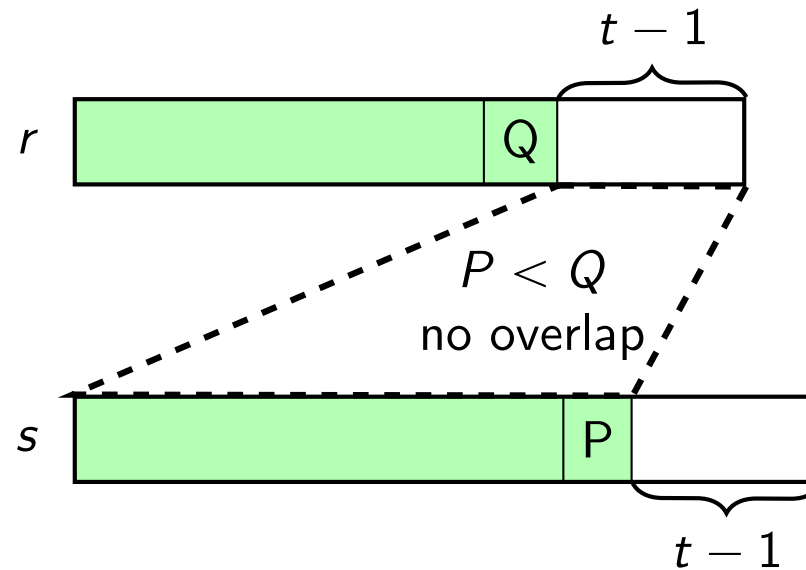
- Consider sets r, s ; Q and P are the largest tokens in the respective **prefixes**, wlog. assume $P < Q$.
- Assume $\text{Pre}(r) \cap \text{Pre}(s) = \emptyset$. We bound $|r \cap s|$:
 - $|\text{Pre}(r) \cap \text{Pre}(s)| = 0$ by assumption
 - $|(r \setminus \text{Pre}(r)) \cap \text{Pre}(s)| = 0$ as $P < Q$
 - $|r \cap (s \setminus \text{Pre}(s))| \leq t-1$ as $|s \setminus \text{Pre}(s)| = t-1$
 - Hence, $|r \cap s| < t$.

Prefix Signature Correctness II



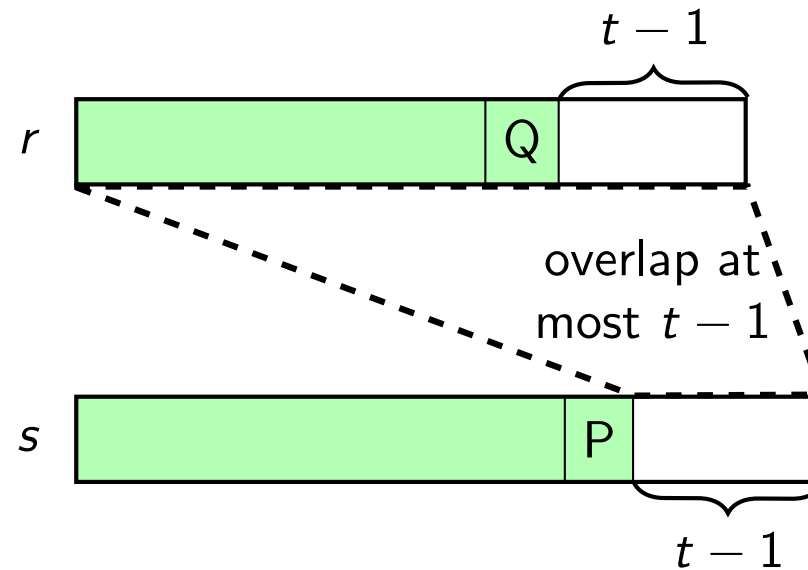
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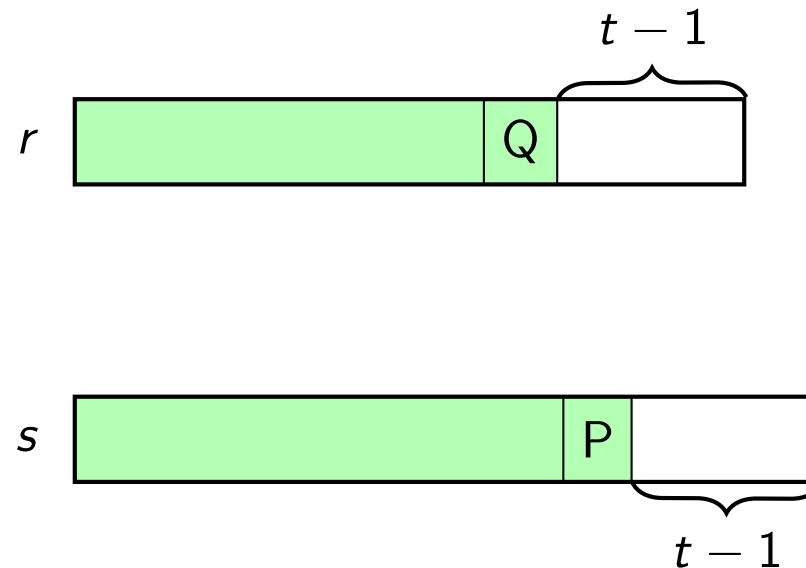
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Prefix Signature Correctness II



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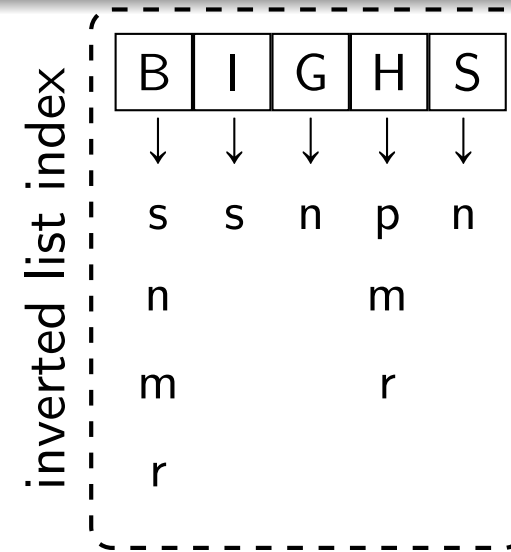
Prefix Signature Correctness II



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 - Hence, $|r \cap s| < t$.

Prefix Signature Example

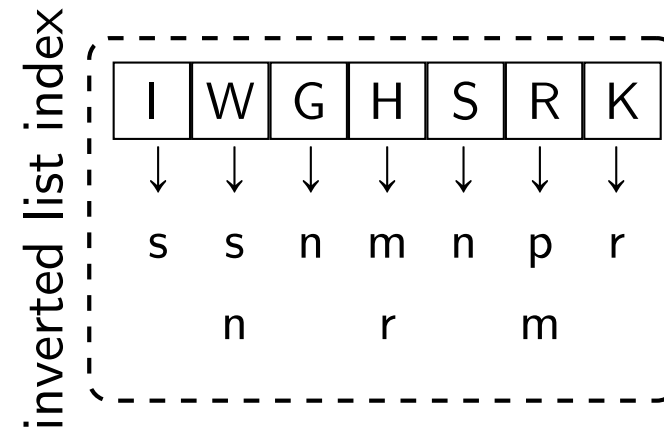
id	interests (ordered alphabetically)
s	{B, I, W}
n	{B, G, S, W}
p	{H, R}
m	{B, H, R}
r	{B, H, K}



- Overlap threshold $t = 2$
- **Indexing:** all tokens except the last, alphabetical order
- **Probing:**
 1. Set s : $C \leftarrow \emptyset \cup \{(s, n), (s, m), (s, r)\}$
 2. Set n : $C \leftarrow C \cup \{(n, m), (n, r)\}$
 3. Set p : $C \leftarrow C \cup \{(p, m), (p, r)\}$
 4. Set m : $C \leftarrow C \cup \{(m, r)\}$
- still 8 comparisons!
- removing B from the prefix could help

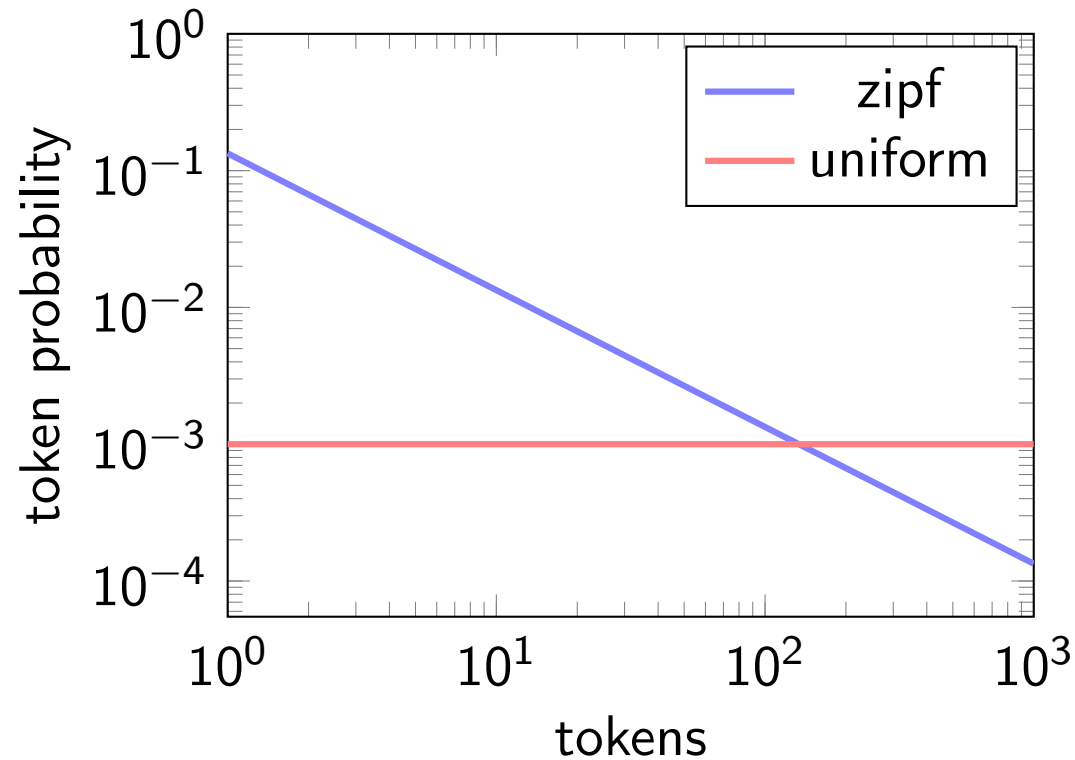
Prefix Signature Example: Order Makes a Difference

id	interests (ordered by frequency)
s	{I, W, B}
n	{G, S, W, B}
p	{R, H}
m	{R, H, B}
r	{K, H, B}



- Overlap threshold $t = 2$
- **Indexing:** all tokens except the last, ordered by ascending frequency
- **Probing:**
 1. Set s : $C \leftarrow \emptyset \cup \{(s, n)\}$
 2. Set n : $C \leftarrow C \cup \emptyset$
 3. Set p : $C \leftarrow C \cup \{(p, m)\}$
 4. Set m : $C \leftarrow C \cup \{(m, r)\}$
- only 3 comparisons!
- **Heuristic:** Ordering by ascending token frequency reduces candidates

Two Distributions



- **Distribution:** Real-world set-data often follow a zipfian distribution
- **Skew:** Some tokens appear frequently, a large number of tokens is uncommon. This favors the prefix signature.

Demonstration

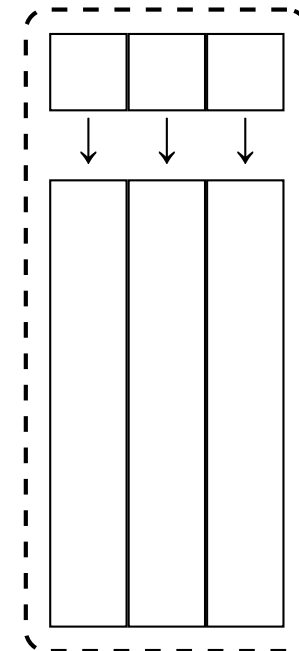
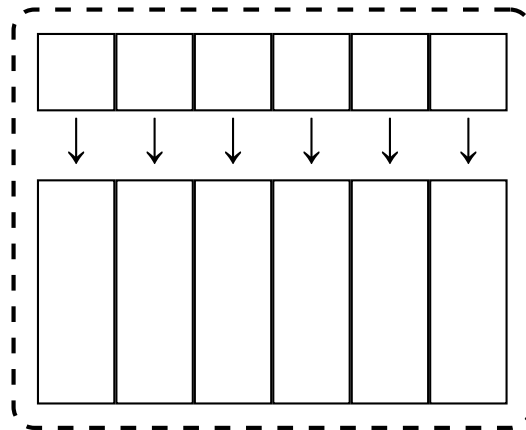
- **Experiment:** Identity signature vs. Prefix signature²
 - self-join with $|R| = 100000$
 - average set size 10
 - universe sizes $|U| = 10000$, uniform and zipfian distribution
 - overlap similarity with threshold $t \in \{4, 6, 8\}$
 - global order: ascending token frequency

Identity				Prefix			
dist.	t	#comp.	runtime [s]	dist.	t	#comp.	runtime [s]
uni.	4	$5.0 \cdot 10^7$	0.187	uni.	4	$2.9 \cdot 10^7$	0.449
	6	$5.0 \cdot 10^7$	0.186		6	$1.8 \cdot 10^7$	0.349
	8	$5.0 \cdot 10^7$	0.186		8	$8.9 \cdot 10^6$	0.145
zipf	4	$3.4 \cdot 10^9$	16.358	zipf	4	$3.1 \cdot 10^8$	3.935
	6	$3.4 \cdot 10^9$	16.862		6	$6.2 \cdot 10^7$	0.873
	8	$3.4 \cdot 10^9$	16.842		8	$1.2 \cdot 10^7$	0.197

²Implementation includes some optimizations compared to framework

Impact of Universe Size

- **Identity and Prefix:** individual tokens used as signatures
- **Runtime:** proportional to sum of all pairs in each list
- **Problem:** small universe size reduces filtering effectiveness



- **Uniform distribution:** halving the universe doubles the list lengths and runtime
- **Idea:** use a more selective signature than individual tokens

Subset Signature I

Lemma

$$\begin{aligned} |r \cap s| &\geq t \\ \Leftrightarrow \\ \exists p \subseteq U : |p| = t \wedge p \subseteq r \wedge p \subseteq s \end{aligned}$$

- Similar sets have at least one common subset of size t . This proves correctness of the following signature:

Definition (Subsets Signature)

The subsets signature $\text{Sub}(r)$ of a set r is defined as:

$$\text{Sub}(r) = \{p \subseteq r \mid |p| = t\}$$

for overlap threshold t .

Subset Signature II

- Sub is stronger than required for signature schemes. It also holds that

$$\text{Sign}(r) \cap \text{Sign}(s) \neq \emptyset \Rightarrow \text{Sim}(r, s) \geq t.$$

Therefore, verification is not necessary.

- For set r and threshold t , we have $|\text{Sub}(r)| = \binom{|r|}{t}$, growing very quickly depending on both $|r|$ and t .
- **Example:** $n = \{B, W, G, S\}$, $t = 2$

$$\begin{aligned} \text{Sub}(n) = & \{\{B, W\}, \{B, G\}, \{B, S\} \\ & \{W, G\}, \{W, S\}, \{G, S\}\} \end{aligned}$$

Demonstration

- **Experiment:** Prefix signature vs. Subsets signature³
 - self-join with $|R| = 1000000$
 - average set size 6
 - universe sizes $|U| \in \{1000, 10000\}$, uniform distribution
 - overlap similarity with threshold $t \in \{3, 4, 5\}$

Prefix			Subsets		
$ U $	t	runtime [s]	$ U $	t	runtime [s]
1000	3	336	1000	3	25.6
	4	233		4	41.0
	5	138		5	42.8
10000	3	40.7	10000	3	32.0
	4	24.7		4	53.9
	5	14.9		5	66.2

- Sub outperforms Pre for small set sizes and small universes
- Pre scales better wrt. set size and threshold for large universes

³Implementation includes some optimizations compared to framework

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Partitioning I

Definition (Partition)

A partition P of universe U is a family of sets $P = \{p_1, \dots, p_n\}$ with the following properties:

1. $\emptyset \notin P$
2. $\bigcup_{p \in P} p = U$
3. For any $p_i, p_j, i \neq j$, we have $p_i \cap p_j = \emptyset$

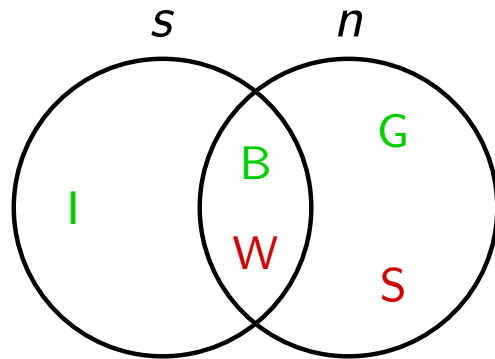
Lemma

For any partition P of universe U and any two sets $r, s \subseteq U$, we have

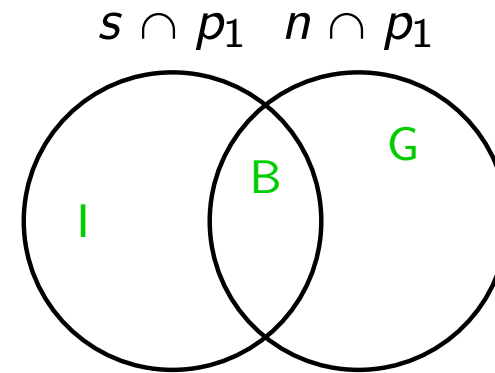
$$\text{Ham}(r, s) = \sum_{p \in P} \text{Ham}(r \cap p, s \cap p)$$

Partitioning II

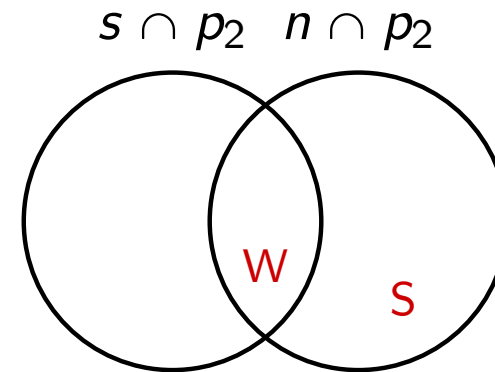
- Example: $P = \{\{B, G, H, I\}, \{K, R, S, W\}\} = \{p_1, p_2\}$



$$|s \Delta n| = 3$$



$$|(s \cap p_1) \Delta (n \cap p_1)| = 2$$



$$|(s \cap p_2) \Delta (n \cap p_2)| = 1$$

Partition Signature

Definition (Partition Signature)

The partition signature $\text{Par}(r)$ of a set r is constructed as follows:

1. Fix any partition P of U into $t + 1$ parts (for Hamming distance t)
2. $\text{Par}(r) = \{(r \cap p_i, i) \mid p_i \in P\}$

- **Example:** $P = \{\{B, G, H, I\}, \{K, R, S, W\}\} = \{p_1, p_2\}$
 - $\text{Par}(\{I, B, W\}) = \{(\{I, B\}, 1), (\{W\}, 2)\}$
 - $\text{Par}(\{S, W\}) = \{(\emptyset, 1), (\{S, W\}, 2)\}$

Correctness of the Partition Signature

Lemma (Correctness of Par)

$$\text{Ham}(r, s) \leq t \Rightarrow \text{Par}(r) \cap \text{Par}(s) \neq \emptyset$$

We show the contraposition $\text{Par}(r) \cap \text{Par}(s) = \emptyset \Rightarrow \text{Ham}(r, s) > t$

Proof.

Assume $\text{Par}(r) \cap \text{Par}(s) = \emptyset$.

- For any $p_i \in P$, if $r \cap p_i \neq s \cap p_i$, then $\text{Ham}(r \cap p_i, s \cap p_i) \geq 1$.
- Hence, $\text{Ham}(r, s) = \sum_{p \in P} \text{Ham}(r \cap p, s \cap p) \geq |P| = t + 1 > t$.



Demonstration

- **Experiment:** Prefix signature⁴ vs. Partition signature
 - self-join with $|R| = 1000000$
 - average set size **20**
 - universe sizes $|U| \in \{1000, 10000\}$, uniform distribution
 - Hamming distance with threshold $t \in \{3, 4, 5\}$

Prefix			Partition		
$ U $	t	runtime [s]	$ U $	t	runtime [s]
1000	3	326	1000	3	4
	4	451		4	23
	5	576		5	144
10000	3	34	10000	3	4
	4	46		4	23
	5	70		5	144

- Par outperforms Pre for large set sizes and small universes
- Par is less sensitive to universe size compared to Pre
- Pre works better for large universes and small set sizes

⁴Adapted to work with Hamming distance

The Empty Par Signature

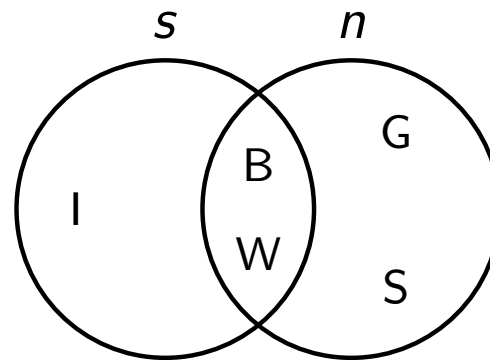
- Consider partitioning $P = \{\{A, B, C\}, \{D, E, F\}, \{G, H, I\}\}$, $t = 2$:

	A	B	C	D	E	F	G	H	I
r	1	1					1	1	
s	1		1				1		1
u		1	1					1	1

- Although all pairs of sets are at Hamming distance 4, they all share the **red** signature.
- Hence, all pairs of sets are candidates!
- This can happen for small sets or heavily skewed distributions.
- More sophisticated partitioning and more flexible searching in each partition can remedy this problem.

Enumeration: Idea

- How can we make s and n the same?



- By removing I from s and both S and G from n .
- **Idea:** By removing at most t tokens, all pairs of sets in Hamming distance t can be made equal.

Enumeration Signature

Definition (Enumeration Signature)

The enumeration signature $\text{En}(r)$ of a set r for Hamming distance with threshold t is given by:

$$\text{En}(r) = \{p \subseteq r \mid |p| \geq |r| - t\}$$

- **Example:** $t = 2$

- $\text{En}(\{I, B, G\}) = \{\{I, B, G\}, \{I, B\}, \{I, G\}, \{B, G\}, \{I\}, \{B\}, \{G\}\}$
- $\text{En}(\{H, R\}) = \{\{H, R\}, \{H\}, \{R\}, \emptyset\}$

Lemma (Correctness of En)

$$\text{Ham}(r, s) \leq t \Rightarrow \text{En}(r) \cap \text{En}(s) \neq \emptyset$$

Correctness of the Enumeration Signature

Proof.

- Assume $\text{Ham}(r, s) = |(r \setminus s) \cup (s \setminus r)| \leq t$.
- Hence, $|r \setminus (r \setminus s)| \geq |r| - t$ and $|s \setminus (s \setminus r)| \geq |s| - t$
- Consider the set $r \setminus (r \setminus s) = r \cap s = s \setminus (s \setminus r)$
- As $r \cap s \subseteq r$ and $|r \cap s| \geq |r| - t$, $r \cap s \in \text{En}(r)$.
- Similarly, $r \cap s \in \text{En}(s)$.
- So $r \cap s \in \text{En}(r) \cap \text{En}(s)$.



Demonstration

- **Experiment:** Partition signature vs. Enumeration signature⁵
 - self-join with $|R| = 1000000$
 - average set size $|\overline{r}| \in \{8, 16\}$
 - universe size $|U| = 10000$, uniform distribution
 - Hamming distance with threshold $t \in \{1, 2, 3\}$

Partition			Enumeration		
$ \overline{r} $	t	runtime [s]	$ \overline{r} $	t	runtime [s]
8	1	4	8	1	12
	2	141		2	42
	3	1034		3	165
16	1	1	16	1	21
	2	3		2	170
	3	14		3	(out of memory)

- En can outperform Par for small thresholds and set sizes
- For large thresholds and sets, En generates too many signatures

⁵Implemented using an optimization called *asymmetric signature scheme* that avoids false positives.

Implementations of Set Similarity Joins

Real implementations of set similarity join algorithms typically

- also support similarity functions other than overlap and Hamming
- use a combination of multiple signature schemes
- extend the algorithmic framework to optimize for their signature schemes
- use additional filters (e.g., based on set *length* or the *positions* of matching signatures)

Outline

1 Filters for the Set Similarity Join

- Motivation
- Signature-based Filtering
- Signatures for Overlap Similarity
- Signatures for Hamming Distance

2 Implementations of Set Similarity Joins

- Other Similarity Functions
- Table of Set Similarity Join Algorithms and their Signatures

3 Conclusion

Other Similarity Functions

- **Normalization:** often, normalized similarity functions are preferred
 - $r = \{A, B, C\}, s = \{A, B\}, u = \{A, B, C, D, E, F, G, H, I\}$
 - The pair (r, u) has higher overlap than the pair (r, s)
 - Still, (r, s) might appear more similar due to fewer different tokens
 - Normalizations also consider set sizes and take values in $[0, 1]$

- **Jaccard:** $\text{Jac}(r, s) = \frac{|r \cap s|}{|r \cup s|} = \frac{|r \cap s|}{|r| + |s| - |r \cap s|}$

- **Dice:** $\text{Dice}(r, s) = \frac{2|r \cap s|}{|r| + |s|}$

- **Cosine:** $\text{Cos}(r, s) = \frac{|r \cap s|}{\sqrt{|r| \cdot |s|}}$

- **Example:**

$$\text{Jac}(r, s) = \frac{|\{A, B\}|}{|\{A, B, C\}|} = \frac{2}{3}$$

$$\text{Jac}(r, u) = \frac{|\{A, B, C\}|}{|\{A, B, C, D, E, F, G, H, I\}|} = \frac{3}{9} = \frac{1}{3}$$

Adapting the Prefix Signature for Jaccard

- The prefix signature operates with overlap similarity
- **Idea:** Bound minimum overlap s.t. two sets r, s can have $\text{Jac}(r, s) \geq t$

$$\begin{aligned} \frac{|r \cap s|}{|r| + |s| - |r \cap s|} &\geq t \\ \Leftrightarrow |r \cap s| &\geq t(|r| + |s| - |r \cap s|) \\ \Leftrightarrow |r \cap s| &\geq \frac{t}{1+t}(|r| + |s|) =: \text{eqo}_J(r, s) \end{aligned}$$

- eqo_J depends on the sizes of two sets. As we want to handle all possible size combinations, we have to get rid of one of them.
- **Possible Solution:** Assume minimal value $|s| = 1$, but better bounds are possible.

Length Bounds for Jaccard

Lemma

If $\text{Jac}(r, s) \geq t$, then $t|r| \leq |s| \leq \frac{|r|}{t}$.

Lemma

If $\text{Jac}(r, s) \geq t$, then

$$\begin{aligned} |r \cap s| &\geq \text{eqo}_J(r, s) \\ &\geq \text{eqo}_J(r, t|r|) \\ &= t|r| \end{aligned}$$

- Hence, for each set r use $\lceil t|r| \rceil$ as the overlap and proceed as in Pre.

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Algorithm	Signature ⁶	Remarks
AllPairs [BMS07]	Length+ Pre	
PPJoin [XWLY08]	Length+ Pre	additional filter based on position of prefix matches
SkipJoin [WQL ⁺ 19]	Length+ Pre	tighter length filter using prefix positions, removes unmatchable entries from index, can leverage knowledge of set similarity “transitively”
SizeAware [DTL18]	Sub (small sets)+ Id (large sets)	avoids enumerating all subsets in Sub by exploiting an order on subsets, handles small and large sets differently
PartEnum [AGK06]	Length+ Par + En	partitions sets into smaller subsets, enumerates in each partition
PartAlloc [DLWF15]	Length+ Par + En	more flexible than PartEnum, has tighter filtering condition
GPH [QXW ⁺ 21]	Par + En	more flexible than PartAlloc, optimizes how partitions are chosen

⁶Refers to the closest signature discussed during the lecture; the listed algorithms often use a more efficient variation of the respective signatures.

Summary

- Naive set similarity join inefficient due to large search space
- Signature-based filters speed up join:
 - Id and Pre: single tokens as signatures
 - Sub: all subsets of overlap size as signatures
 - Par: non-overlapping subsets as signatures
 - En: all subsets in Hamming distance as signatures
- Performance depends on dataset's characteristics



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