

Similarity Search

Trees

Nikolaus Augsten

nikolaus.augsten@plus.ac.at
Department of Computer Science
University of Salzburg



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Outline

- 1 What is a Tree?
- 2 Encoding XML as Trees

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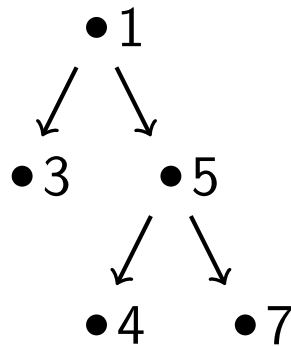
- 1 What is a Tree?
- 2 Encoding XML as Trees

What is a Tree?

- **Graph**: a pair (N, E) of nodes N and edges E between nodes of N
- **Tree**: a directed, acyclic graph T
 - that is connected and
 - no node has more than one incoming edge
- **Edges**: $E(T)$ are the edges of T
 - an edge $(p, c) \in E(T)$ is an ordered pair
 - with $p, c \in N(T)$
- **“Special” Nodes**: $N(T)$ are the nodes of T
 - **parent/child**: $(p, c) \in E(T) \Leftrightarrow p$ is the parent of c , c is the child of p
 - **siblings**: c_1 and c_2 are siblings if they have the same parent node
 - **root node**: node without parent (no incoming edge)
 - **leaf node**: node without children (no outgoing edge)
 - **fanout**: fanout f_v of node v is the number of children of v

Unlabeled Trees

- Unlabeled Tree:
 - the focus is on the structure, not on distinguishing nodes
 - however, we need to distinguish nodes in order to define edges
 $\Rightarrow$ each node v has a unique identifier $\text{id}(v)$ within the tree
- Example: $T = (\{1, 3, 5, 4, 7\}, \{(1, 3), (1, 5), (5, 4), (5, 7)\})$



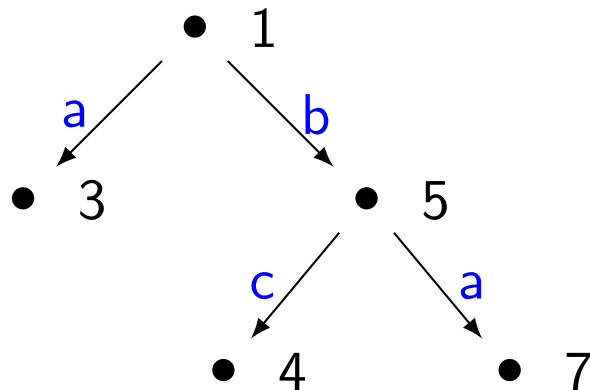
Edge Labeled Trees

- **Edge Labeled Tree:**

- an edge $e \in E(T)$ between nodes a and b is a triple $e = (\text{id}(a), \text{id}(b), \lambda(e))$
- $\text{id}(a)$ and $\text{id}(b)$ are node IDs
- $\lambda(e)$ is the edge label (not necessarily unique within the tree)

- **Example:**

$$T = (\{1, 3, 5, 4, 7\}, \{(1, 3, a), (1, 5, b), (5, 4, c), (5, 7, a)\})$$



Node Labeled Trees

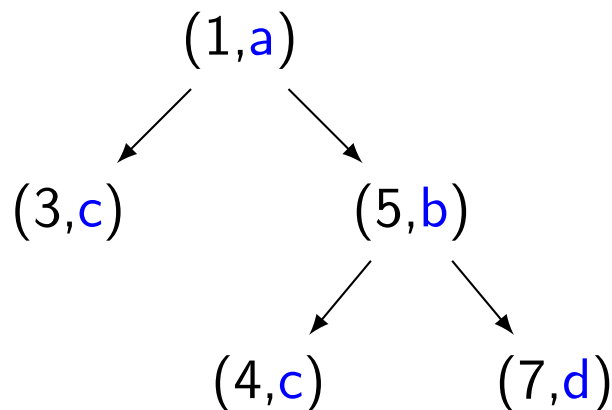
- **Node Labeled Tree:**

- a node $v \in N(T)$ is a pair $(\text{id}(v), \lambda(v))$
- $\text{id}(v)$ is unique within the tree
- label $\lambda(v)$ needs not to be unique

- **Intuition:**

- The identifier is the key of the node.
- The label is the data carried by the node.

- **Example:** $T = (\{(1, a), (3, c), (5, b), (4, c), (7, d)\}, \{(1, 3), (1, 5), (5, 4), (5, 7)\})$



Notation and Graphical Representation

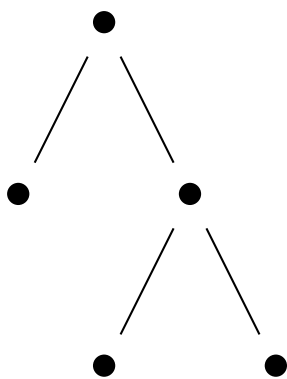
- Notation:

- node identifiers: $\text{id}(v_i) = i$
- tree identifiers: $T_1, T_2, \dots$

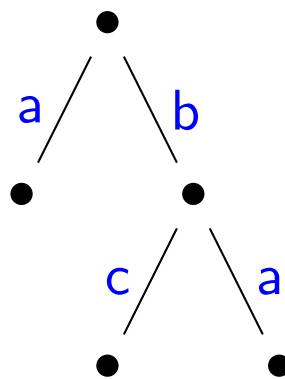
- Graphical representation

- we omit brackets for (identifier,label)-pairs
- we (sometimes) omit node identifiers at all
- we do not show the direction of edges
(edges are always directed from root to leave)

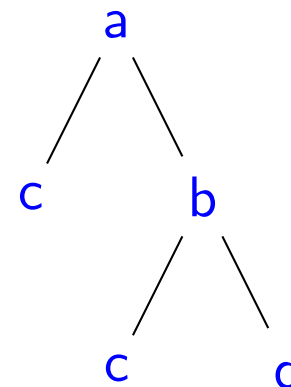
unlabeled tree



edge labeled tree



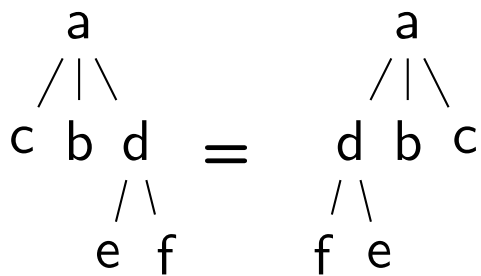
node labeled tree



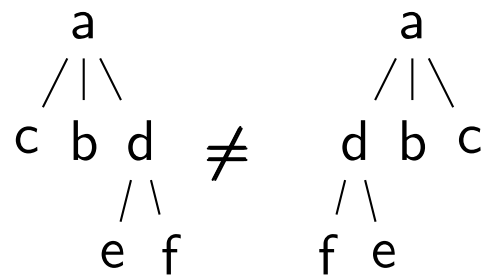
Ordered Trees

- **Ordered Trees:** siblings are ordered
- **contiguous** siblings $s_1 < s_2$ have no sibling x such that $s_1 < x < s_2$
- c_i is the **i -th child** of p if
 - p is the parent of c_i , and
 - $i = |\{x \in N(T) : (p, x) \in E(T), x \leq c_i\}|$
- **Example:**

Unordered Trees



Ordered Trees

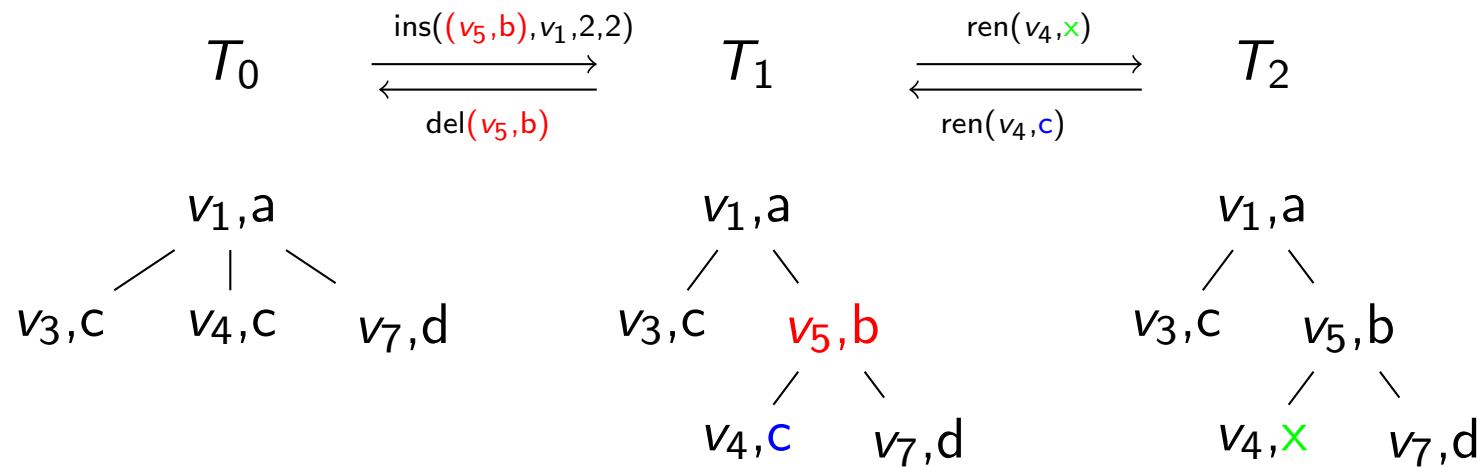


- **Note:** “ordered” does not necessarily mean “sorted alphabetically”

Edit Operations

- We assume **ordered, labeled trees**
- **Rename node**: $ren(v, l')$
 - change label l of v to $l' \neq l$
- **Delete node**: $del(v)$ (v is not the root node)
 - remove v
 - connect v 's children directly to v 's parent node (preserving order)
- **Insert node**: $ins(v, p, k, m)$
 - detach m consecutive children of p , starting with the child at position k , i.e., the children $c_k, c_{k+1}, \dots, c_{k+m-1}$
 - attach $c_k, c_{k+1}, \dots, c_{k+m-1}$ as children of the new node v (preserving order)
 - insert new node v as k -th child of p
- Insert and delete are **inverse** edit operations (i.e., insert undoes delete and vice versa)

Example: Edit Operations



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Representing XML as a Tree

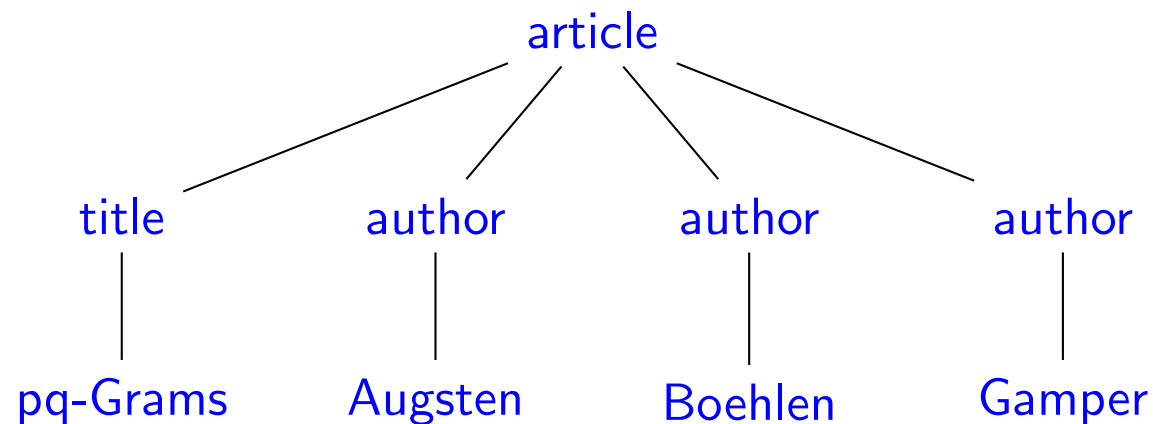
- Many possibilities – we will consider
 - single-label tree
 - double-label tree
- Pros/cons depend on application!

XML as a Single-Label Tree

- The XML document is encoded as a tree with:
 - XML element: node labeled with element tag name
 - XML attribute: node labeled with attribute name
 - Text contained in elements/attributes: node labeled with the text-value
- Element nodes contain:
 - nodes of their sub-elements
 - nodes of their attributes
 - nodes with their text values
- Attribute nodes contain:
 - single node with their text value
- Text nodes are always leaves
- Order:
 - sub-element and text nodes are ordered
 - attributes are not ordered (approach: store them before all sub-elements, sort according to attribute name)

Example: XML as a Single-Label Tree

```
<article title='pq-Grams'>  
  <author>Augsten</author>  
  <author>Boehlen</author>  
  <author>Gamper</author>  
</article>
```

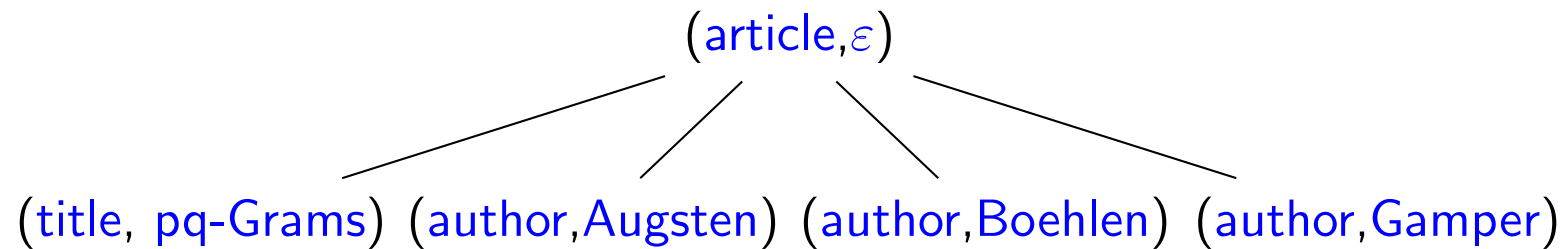


XML as a Double-Label Tree

- Node labels are pairs
- The XML document is encoded as a tree with:
 - XML element: node labeled with (tag-name,text-value)
 - XML attribute: node labeled with (attribute-name,text-value)
- Element nodes contain:
 - nodes of their sub-elements and attributes
- Attribute nodes are always leaves
- Element nodes without attributes or sub-elements are leaves
- Order:
 - sub-element nodes are ordered
 - attributes are not ordered (approach: see previous slide)
- Limitation: Can represent
 - *either* elements with sub-elements and/or attributes
 - *or* elements with a text value

Example: XML as a Double-Label Tree

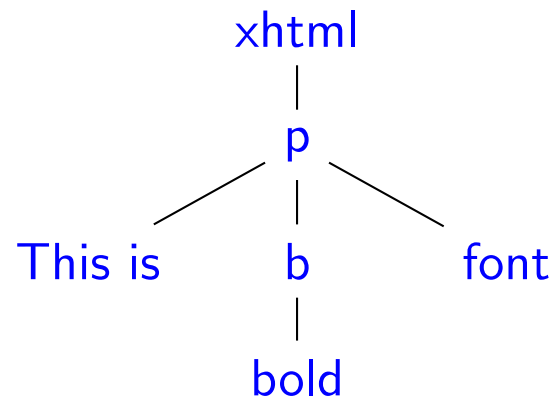
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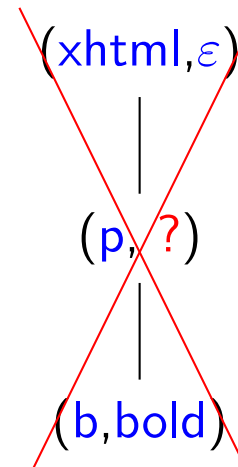
Example: Single- vs. Double-Label Tree

```
<xhtml>
  <p>This is <b>bold</b> font.</p>
</xhtml>
```

Single-Label Tree



Double-Label Tree



Parsing XML

We discuss two popular parsers for XML:

- DOM – Document Object Model
- SAX – Simple API for XML
- StAX – Streaming API for XML

DOM – Document Object Model

- W3C¹ standard for accessing and manipulating XML documents
- Tree-based: represents an XML document as a tree (single-label tree with additional node info, e.g. node type)
- Elements, attributes, and text values are nodes
- DOM parsers load XML into main memory
 - random access by traversing tree :-)
 - large XML documents do not fit into main memory :-)

¹<http://www.w3schools.com/dom>

SAX / StAX

- **SAX** – Simple API for XML²
 - ‘de facto’ standard for parsing XML
- Event-based: reports parsing events (e.g., start and end of elements)
 - no random access :-)
 - you see only one element/attribute at a time
 - you can parse (arbitrarily) large XML documents :-)
- **StAX** - Streaming API for XML³
 - similar to SAX, but pull-based (vs. SAX: push)
 - pull: the client receives the next event on request
- Java API available for DOM, SAX, and StAX.
- For importing XML into a database: **use SAX or StAX!**

²<http://www.saxproject.org>

³<https://en.wikipedia.org/wiki/StAX>